| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **8(a)** | $(2r-1)^2 = 4r^2 - 4r + 1$ | B1 | 1.1b |
| | $\sum_{r=1}^{n}(2r-1)^2 = 4\sum_{r=1}^{n}r^2 - 4\sum_{r=1}^{n}r + \sum_{r=1}^{n}1$<br>$= 4\frac{n}{6}(n+1)(2n+1) - 4\frac{n}{2}(n+1) + n$ | M1<br>A1 | 1.1b<br>1.1b |
| | $= \frac{n}{3}\left[2(n+1)(2n+1) - 6(n+1) + 3\right]$<br>Or<br>$= n\left[\frac{2}{3}(n+1)(2n+1) - 2(n+1) + 1\right]$ | dM1 | 1.1b |
| | $\left\{\frac{n}{3}\left(4n^2 + 6n + 2 - 6n - 6 + 3\right)\right\}$<br>$= \frac{n}{3}\left(4n^2 - 1\right)$ cso | A1 | 2.1 |
| | | **(5)** | |
| **(b)** | $\sum_{r=51}^{500}(2r-1)^2$ | B1 | 3.1a |
| | $\sum_{r=51}^{500}(2r-1)^2 = \sum_{r=1}^{500}(2r-1)^2 - \sum_{r=1}^{50}(2r-1)^2$<br>$= \frac{500}{3}\left(4(500)^2 - 1\right) - \frac{50}{3}\left(4(50)^2 - 1\right)$<br>$\{ = 166666500 - 166650\}$ | M1 | 1.1b |
| | 166 499 850 | A1 | 1.1b |
| | | **(3)** | |
| | | | **(8 marks)** |

**Notes:**

**(a)**

**B1:** Correct expanded expression.

**M1:** Substitutes at least one of the standard formulae into their expanded expression.

**A1:** Fully correct unsimplified expression.

**dM1:** Dependent on previous method. Attempts to factorises out *n*. Must have a *n* in every term. Condone a slip with one term as long as the intention is clear.

**A1:** Achieves the correct answer, with a correct intermediate line of working. cso

**Note** If uses $\sum 1 = 1$ scores B1 M1 A0 M0 A0

An attempt at proof by induction may score B1 only

**(b)**

**B1:** Correct summation formula for the sum of the squares of all positive odd three-digit integers including limits. This can be implied by later work.

**M1:** Uses the answer to part (a) and $\sum_{r=p}^{q}(2r-1)^2=\sum_{r=1}^{q}(2r-1)^2-\sum_{r=1}^{p-1}(2r-1)^2$ where $p$, $q$ are numerical and $q>p$, to find a value. There must be some indication of the sum that they are finding or the correct values for $p$ and $q$.

States $\sum_{r=1}^{500}(2r-1)^2-\sum_{r=1}^{50}(2r-1)^2$ implies B1

States $\frac{500}{3}\left(4(500)^2-1\right)-\frac{50}{3}\left(4(50)^2-1\right)$ this scores B1 (implied) and M1

**A1:** Correct value

**Note** $\sum_{r=51}^{500}(2r-1)^2=166499850$ or correct answer only scores B1 M0 A0, must be evidence of using the answer to (a)