Question	Scheme	Marks	AOs
9(i)	$\begin{vmatrix} k & -2 & 7 \\ -3 & -5 & 2 \\ k & k & 4 \end{vmatrix} = k(-20-2k)+2(-12-2k)+7(-3k+5k)$ or $\begin{vmatrix} k & -2 & 7 & k & -2 \\ -3 & -5 & 2 & -3 & -5 \\ k & k & 4 & k & k \end{vmatrix} = k(-5)(4)-2(2)(k)+7(-3)(k)$ $-7(-5)(k)-k(2)(k)-(-2)(-3)(4)$	M1	1.1b
	$-2k^2-10k-24(=0)$ isw	A1	1.2
	$b^2-4ac=(10)^2-4(-2)(-24)=\ldots$ $b^2-4ac=(5)^2-4(-1)(-12)=\ldots$ Or $k^2+5k+12=0 \Rightarrow (k+2.5)^2+5.75=0 \Rightarrow (k+2.5)^2=-5.75$ $-2k^2-10k-25=0 \Rightarrow -2(k+2.5)^2-12.5=0 \Rightarrow (k+2.5)^2=-5.75$ Or $k^2+5k+12 \Rightarrow (k+2.5)^2+5.75 \Rightarrow (k+2.5)^2 \geqslant 0$ or $-2k^2-10k-25=0 \Rightarrow -2(k+2.5)^2-12.5=0 \Rightarrow -2(k+2.5)^2 \leqslant 0$ Or $\frac{\mathrm{d}(-2k^2-10k-24)}{\mathrm{d}k}=-4k-10=0 \Rightarrow k=-2.5 \Rightarrow \text{determinant}=-5.75$ Or $k=\frac{10\pm\sqrt{(-10)^2-4(-2)(-25)}}{2(-2)}=\frac{-5\pm\sqrt{23}\mathrm{i}}{2}$	M1	1.1b
	$b^2-4ac=-92<0$ therefore **no real roots** so **non-singular** $b^2-4ac=-23<0$ therefore **no real roots** so **non-singular** Or Square of negative is not real therefore **non-singular** Or $(k+2.5)^2+5.75>0$ therefore **no real roots** so **non-singular** $-2(k+2.5)^2-12.5<0$ therefore **no real roots** so **non-singular** Or As negative quadratic maximum value of determinant = – 5.25 therefore **no real roots** so **non-singular** Or Imaginary roots therefore **no real roots** so **non-singular**	A1	2.4
		(4)	

(ii)	$\begin{pmatrix}2 & -1\\-3 & 0\end{pmatrix}\begin{pmatrix}a & 4\\2 & -a\end{pmatrix}=\begin{pmatrix}\dots & \dots\\\dots & \dots\end{pmatrix}$ can be done separately for each point	M1	3.1a
	$\begin{pmatrix}2a-2 & 8+a\\-3a & -12\end{pmatrix}$ or $(2a-2,\ -3a)$ and $(8+a,-12)$	A1	1.1b
	$\sqrt{\left[(2a-2)-(8+a)\right]^2+\left[-3a-(-12)\right]^2}=\sqrt{58}$ or $\overrightarrow{AB}=\begin{pmatrix}8+a\\-12\end{pmatrix}-\begin{pmatrix}2a-2\\-3a\end{pmatrix}=\begin{pmatrix}10-a\\-12+3a\end{pmatrix}$ or $\overrightarrow{BA}=\begin{pmatrix}2a-2\\-3a\end{pmatrix}-\begin{pmatrix}8+a\\-12\end{pmatrix}=\begin{pmatrix}a-10\\12-3a\end{pmatrix}$ $(a-10)^2+(12-3a)^2=58$ or $(10-a)^2+(3a-12)^2=58$ leading to a 3TQ	M1	3.1a
	$10a^2-92a+186=0$	A1	1.1b
	$a=3, \frac{31}{5}$ o.e. cso	A1	1.1b
		(5)	

(9 marks)

Notes:

(i)

M1: Correct method to find the determinant, condone a single sign slip but not on second term must be +2 (…)

Note: May expand along any row or column.

A1: Correct simplified determinant

M1: Either

- Finds the value of the discriminant or sufficient working seen to identify the sign e.g. 100 – 192
- Completes the square an rearranges so that $(k\pm a)^2=-b$
- Completes the square and states that $(k\pm a)^2\geqslant 0$
- Completes the square and states that $-\alpha(k\pm a)^2\leqslant 0$
- Differentiates the determinant to find the coordinates of the vertex
- Use the quadratic formula to find the imaginary roots

A1: Correct solution only

Either

- Correct value for the discriminant (may be implied), concludes less than 0, therefore no real roots and non singular.
- Correct completing the square and conclude no real roots as square root of negative therefore non singular
- Correct completing the square and shows > 0 therefore no real roots and non singular.
- Correct completing the square and shows < 0 therefore no real roots and non singular.

- Correct coordinates of the vertex and negative quadratic therefore no real roots and non singular.
- Use the quadratics formula to find the correct imaginary roots therefore no real roots/value for k and non singular.

Note $k = \dfrac{-5 \pm \sqrt{23}\mathrm{i}}{2}$ which is not real is M0 A0 unless uses the quadratic formula or completing the square to show where this has come from

(ii)

M1: Uses matrix **Q** to find the coordinates of the points A' and B'. Condone a sign slip.

A1: Correct coordinates for the points A' and B', they do not need to be labelled

M1: Finds the distance between their points A' and B' which must not be equal to A and B, sets equal to $\sqrt{58}$, forms a 3TQ.

A1: Correct 3TQ form correct coordinates

A1: Correct values cso

Misread: A common misread is 3 instead of – 3, the first 3 mark only can be scored using the misread rule

M1: $\begin{pmatrix} 2 & -1 \\ 3 & 0 \end{pmatrix}\begin{pmatrix} a & 4 \\ 2 & -a \end{pmatrix} = \begin{pmatrix} \dots & \dots \\ \dots & \dots \end{pmatrix}$

A1: $\begin{pmatrix} 2a-2 & 8+a \\ 3a & 12 \end{pmatrix}$ or $(2a-2,\ 3a)$ and $(8+a, 12)$

M1: $\sqrt{\left[(2a-2)-(8+a)\right]^2 + \left[3a-12\right]^2} = \sqrt{58}$

A0, A0 This does lead to the correct answer but can score the first three marks only.