Question	Scheme	Marks	AOs
10(i)	$w = 3z - 1 \Rightarrow z = \frac{w + 1}{3}$	B1	3.1a
	$\left(\frac{w+1}{3}\right)^4 + 5\left(\frac{w+1}{3}\right)^2 - 30 = 0$	M1	3.1a
	$\frac{1}{81}\left(w^{4} + 4w^{3} + 6w^{2} + 4w + 1\right) + \frac{5}{9}\left(w^{2} + 2w + 1\right) - 30 = 0$ leading to $w^{4} + aw^{3} + bw^{2} + cw + d \{=0\}$	M1	1.1b
	$w^{4} + 4w^{3} + 51w^{2} + 94w - 2384 = 0$	A1 A1	1.1b 1.1b
		(5)	
	Alternative $p+q+r+s=0$, $pq+pr+ps+qr+qs+rs=5$ $pqr+pqs+prs+qrs=0$, $pqrs=-30$	B1	3.1a
	New sum = $3(p+q+r+s)-4 =\{-4\}$ New pair sum = $9(pq+pr+ps+qr+qs+rs)-9(p+q+r+s)+6 =\{51\}$ New triple sum = $27(pqr+pqs+prs+qrs)-18(pq+pr+ps+qr+qs+rs)$ + $6(p+q+r+s)-4 =\{-94\}$ = $81(pqrs)-27(pqr+pqs+prs+qrs)$ New product + $9(pq+pr+ps+qr+qs+rs)-3(p+q+r+s)+1$ = $\{-2384\}$	M1	3.1a
	Applies $w^4 - (\text{new sum})w^3 + (\text{new pair sum})w^2 - (\text{new triple sum})w + (\text{new product}) = 0$	M1	1.1b
	$w^4 + 4w^3 + 51w^2 + 94w - 2384 = 0$	A1 A1	1.1b 1.1b
		(5)	
(ii) (a)	$\alpha + 2\alpha + \alpha - \beta = 0$ and $\alpha \times 2\alpha \times (\alpha - \beta) = -\frac{81}{4}$	M1 A1	3.1a 1.1b
	Solves simultaneously e.g. $4\alpha - \beta = 0 \Rightarrow \beta = 4\alpha$ $2\alpha^2 (\alpha - 4\alpha) = -\frac{81}{4} \Rightarrow \alpha^3 = \frac{27}{8} \Rightarrow \alpha =$	M1	3.1a
	Uses their values $\alpha = \frac{3}{2}\beta = 6$ to find the roots α , 2α , $\alpha - \beta$	M1	1.1b
	Roots 1.5, 3, – 4.5	A1	1.1b

		(5)		
(ii) (b)	$n = [(1.5 \times 3) + (1.5 \times -4.5) + (3 \times -4.5)] \times 4$			
	Or			
	Multiplies out $(x-3)\left(x-\frac{3}{2}\right)\left(x+\frac{9}{2}\right)$ or $(x-3)(2x-3)(2x+9)$ to	M1	1.1b	
	achieve the form $4x^3 + \dots$			
	$n = -63 \operatorname{cso}$ (must have correct roots in (a)	A1	1.1b	
		(2)		
		(12 marks)		

Notes:

(i)

B1: Selects the method of making a connection between *z* and *w* by writing $z = \frac{w+1}{3}$. Other variables

may be used

M1: Applies the process of substituting their $z = \frac{w+1}{3}$ into $z^4 + 5z^2 - 30 = 0$

M1: Manipulates their equation into the form $w^4 + aw^3 + bw^2 + cw + d = 0$ having substituted their z in terms of w. Note that the "= 0" can be missing for this mark.

A1: At least two of a, b, c, d correct. Note that the "= 0" can be missing for this mark.

A1: Fully correct equation including "= 0" Must be in terms of w

(i) Alternative

B1: Selects the method of giving four correct equations containing p, q, r and s

M1: Applies the process of finding **at least three** of the new sum, new pair sum, new triple sum and new product. Condone slips but the intention is clear and uses their values.

M1: Applies $w^4 - (\text{new sum})w^3 + (\text{new pair sum})w^2 - (\text{new triple sum})w + (\text{new product}) = 0$.

Condone use of any variable for this mark.

Note that the "= 0" can be missing for this mark.

A1: At least two of a, b, c, d correct. Note that the "= 0" can be missing for this mark.

A1: Fully correct equation including "= 0" Must be in terms of w

(ii) (a)

M1: Uses the sum and product to form two equations in α and β . Condone product = $\frac{81}{4}$ for this mark

Note:
$$4\alpha - \beta = -\frac{n}{4}$$
 or $4\alpha - \beta = 81$ is M0

A1: Correct equations need not be simplied

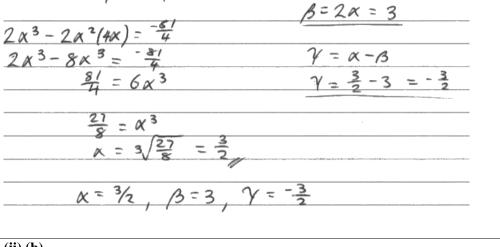
M1: Solves simultaneous equations to find a value for α or β

M1: Uses their values for α and β to find the roots using α , 2α , $\alpha - \beta$. Condone third root as β

A1: Correct roots

Candidates may use their own notation for the roots

Condone confusion with β for M1 but A0



(ii) (b)

M1: Finds the pair sum for their numerical roots and multiplies by 4

Alternative multiplies out three brackets $(x - \text{their } \alpha)(x - \text{their } 2\alpha)(x - (\alpha - \beta))$ to achieve the form $4x^3 + \dots$

A1: Correct value from correct roots only