Question	Scheme	Marks	AOs
10(i)	$w = 3z - 1 \Rightarrow z = \dfrac{w+1}{3}$	B1	3.1a
	$\left(\dfrac{w+1}{3}\right)^4 + 5\left(\dfrac{w+1}{3}\right)^2 - 30 = 0$	M1	3.1a
	$\dfrac{1}{81}\left(w^4 + 4w^3 + 6w^2 + 4w + 1\right) + \dfrac{5}{9}\left(w^2 + 2w + 1\right) - 30 = 0$ leading to $w^4 + aw^3 + bw^2 + cw + d$ $\{= 0\}$	M1	1.1b
	$w^4 + 4w^3 + 51w^2 + 94w - 2384 = 0$	A1 A1	1.1b 1.1b
		(5)	
	Alternative $p + q + r + s = 0, \quad pq + pr + ps + qr + qs + rs = 5$ $pqr + pqs + prs + qrs = 0, \quad pqrs = -30$	B1	3.1a
	New sum $= 3(p+q+r+s) - 4 = \ldots\{-4\}$ New pair sum $= 9(pq + pr + ps + qr + qs + rs) - 9(p+q+r+s) + 6 = \ldots\{51\}$ New triple sum $= 27(pqr + pqs + prs + qrs) - 18(pq + pr + ps + qr + qs + rs)$ $+6(p+q+r+s) - 4 = \ldots\{-94\}$ $= 81(pqrs) - 27(pqr + pqs + prs + qrs)$ New product $+9(pq + pr + ps + qr + qs + rs) - 3(p+q+r+s) + 1$ $= \ldots\{-2384\}$	M1	3.1a
	Applies $w^4 - (\text{new sum})w^3 + (\text{new pair sum})w^2 - (\text{new triple sum})w$ $+ (\text{new product}) = 0$	M1	1.1b
	$w^4 + 4w^3 + 51w^2 + 94w - 2384 = 0$	A1 A1	1.1b 1.1b
		(5)	
(ii) (a)	$\alpha + 2\alpha + \alpha - \beta = 0$ and $\alpha \times 2\alpha \times (\alpha - \beta) = -\dfrac{81}{4}$	M1 A1	3.1a 1.1b
	Solves simultaneously e.g. $4\alpha - \beta = 0 \Rightarrow \beta = 4\alpha$ $2\alpha^2(\alpha - 4\alpha) = -\dfrac{81}{4} \Rightarrow \alpha^3 = \dfrac{27}{8} \Rightarrow \alpha = \ldots$	M1	3.1a
	Uses their values $\alpha = \dfrac{3}{2}\ \beta = 6$ to find the roots $\alpha, 2\alpha, \alpha - \beta$	M1	1.1b
	Roots 1.5, 3, – 4.5	A1	1.1b

		(5)	
(ii) (b)	$n=\left[(1.5\times3)+(1.5\times-4.5)+(3\times-4.5)\right]\times4$ Or Multiplies out $(x-3)\left(x-\frac{3}{2}\right)\left(x+\frac{9}{2}\right)$ or $(x-3)(2x-3)(2x+9)$ to achieve the form $4x^3+...$	M1	1.1b
	$n=-63$ cso (must have correct roots in (a)	A1	1.1b
		(2)	
			(12 marks)

Notes:

(i)

B1: Selects the method of making a connection between z and w by writing $z=\frac{w+1}{3}$. Other variables may be used

M1: Applies the process of substituting their $z=\frac{w+1}{3}$ into $z^4+5z^2-30=0$

M1: Manipulates their equation into the form $w^4+aw^3+bw^2+cw+d=0$ having substituted their z in terms of w. Note that the "= 0" can be missing for this mark.

A1: At least two of a, b, c, d correct. Note that the "= 0" can be missing for this mark.

A1: Fully correct equation including "= 0" Must be in terms of w

(i) Alternative

B1: Selects the method of giving four correct equations containing p, q, r and s

M1: Applies the process of finding **at least three** of the new sum, new pair sum, new triple sum and new product. Condone slips but the intention is clear and uses their values.

M1: Applies $w^4-(\text{new sum})w^3+(\text{new pair sum})w^2-(\text{new triple sum})w+(\text{new product})=0$.

Condone use of any variable for this mark.

Note that the "= 0" can be missing for this mark.

A1: At least two of a, b, c, d correct. Note that the "= 0" can be missing for this mark.

A1: Fully correct equation including "= 0" Must be in terms of w

(ii) (a)

M1: Uses the sum and product to form two equations in α and β. Condone product = $\frac{81}{4}$ for this mark

Note: $4\alpha-\beta=-\frac{n}{4}$ or $4\alpha-\beta=81$ is M0

A1: Correct equations need not be simplied

M1: Solves simultaneous equations to find a value for α or β

M1: Uses their values for α and β to find the roots using $\alpha,\ 2\alpha,\ \alpha-\beta$. Condone third root as β

A1: Correct roots

Candidates may use their own notation for the roots

Condone confusion with β for M1 but A0

$$2\alpha^3 - 2\alpha^2(4\alpha) = \frac{-81}{4}$$

$$2\alpha^3 - 8\alpha^3 = \frac{-81}{4}$$

$$\frac{81}{4} = 6\alpha^3$$

$$\frac{27}{8} = \alpha^3$$

$$\alpha = \sqrt[3]{\frac{27}{8}} = \frac{3}{2}$$

$$\beta = 2\alpha = 3$$

$$\gamma = \alpha - \beta$$

$$\gamma = \frac{3}{2} - 3 = -\frac{3}{2}$$

$$\alpha = 3/2,\ \beta = 3,\ \gamma = \frac{-3}{2}$$

(ii) (b)

M1: Finds the pair sum for their numerical roots and multiplies by 4

Alternative multiplies out three brackets $(x - \text{their } \alpha)(x - \text{their } 2\alpha)(x - (\alpha - \beta))$ to achieve the form $4x^3 + \ldots$

A1: Correct value **from correct roots only**