

Question	Scheme	Marks	AOs
2(a)	$\alpha = 210$	B1	1.1b
		(1)	
(b)	Require $k \times$ their '210' divisible by 360	M1	1.1b
	$k = 12$	A1	1.1b
		(2)	
(c)	$\{\mathbf{N}=\} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	B1	1.1b
		(1)	
(d)	$\mathbf{MA} = \begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -2 \\ 4 \\ 3 \end{pmatrix} = \begin{pmatrix} \sqrt{3}+2 \\ 1-2\sqrt{3} \\ 3 \end{pmatrix}$	M1	1.1a
	$\mathbf{NMA} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \sqrt{3}+2 \\ 1-2\sqrt{3} \\ 3 \end{pmatrix} = \begin{pmatrix} \sqrt{3}+2 \\ 2\sqrt{3}-1 \\ 3 \end{pmatrix} *$		
	<p style="text-align: center;">Or</p> $\mathbf{NM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$		
	$\mathbf{NMA} = \begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -2 \\ 4 \\ 3 \end{pmatrix} = \begin{pmatrix} \sqrt{3}+2 \\ 2\sqrt{3}-1 \\ 3 \end{pmatrix} *$		
	i.e. $B(2+\sqrt{3}, 2\sqrt{3}-1, 3) *$	A1*	1.1b
		(2)	

(e)	$AB^2 = OA^2 + OB^2 - 2OA \cdot OB \cos AOB$ $\Rightarrow (4 + \sqrt{3})^2 + (5 - 2\sqrt{3})^2 = 29 + 29 - 2\sqrt{29} \cdot \sqrt{29} \cos AOB$ <p style="text-align: center;">or</p> $OA \cdot OB = \begin{pmatrix} -2 \\ 4 \\ 3 \end{pmatrix} \bullet \begin{pmatrix} 2 + \sqrt{3} \\ 2\sqrt{3} - 1 \\ 3 \end{pmatrix} = 1 + 6\sqrt{3} = \sqrt{29} \cdot \sqrt{29} \cos AOB$	M1	3.1a
	$AOB = 66.9^\circ *$	A1*	1.1b
		(2)	
(f)	$\text{Area } AOB = \frac{1}{2} \sqrt{29} \sqrt{29} \sin 66.9^\circ$ <p>You may see this outside spec from candidates studying 8FM0 21</p> $\frac{1}{2} \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 4 & 3 \\ 2 + \sqrt{3} & 2\sqrt{3} - 1 & 3 \end{vmatrix}$ $= \frac{1}{2} \left[12 - 3(2\sqrt{3} - 1) \right] \mathbf{i} - \left[-6 - 3(2 + \sqrt{3}) \right] \mathbf{j} + \left[-2(2\sqrt{3} - 1) - 4(2 + \sqrt{3}) \right] \mathbf{k}$ $= \frac{1}{2} \left[(15 - 6\sqrt{3}) \mathbf{i} + (12 + 3\sqrt{3}) \mathbf{j} + (-6 - 8\sqrt{3}) \mathbf{k} \right]$ $= \frac{1}{2} \sqrt{(15 - 6\sqrt{3})^2 + (12 + 3\sqrt{3})^2 + (-6 - 8\sqrt{3})^2}$	M1	1.1b
	= 13.3 cao	A1	1.1b
		(2)	

(10 marks)

Notes

- (a)
B1: Correct value, check within the question. If more than one value is stated the correct value must clearly be selected.
- (b)
M1: Uses their answer from part (a) to determine a value for k so that $k \times$ their 210 is divisible by 360. If their answer to part (a) is in radians $k \times$ their $\frac{7\pi}{6}$ is divisible by 2π .
- A1: Correct value must be using an angle of 210. There may be no working but allow M1 A1 for $k = 12$ following an answer of 210 or $\frac{7\pi}{6}$ in part (a)
- Note: an angle of 30, 150, 330 also gives $k = 12$ but is M1A0**
- (c)
B1: Correct matrix
- (d)
M1: Complete method to find the coordinates of B . Look for at least two correct terms for each multiplication, follow through when multiplying by \mathbf{N} .
Alternatively finds \mathbf{NM} (not \mathbf{MN}), look for 4 correct non zero terms if no method is shown and then multiplies to find the coordinates of B .

A1*: Correct coordinates (condone vector) Must have been working with exact values throughout. If working in decimals M1 A0

It is insufficient to just write $\mathbf{NM} \begin{pmatrix} -2 \\ 4 \\ 3 \end{pmatrix} = \begin{pmatrix} \sqrt{3} + 2 \\ 2\sqrt{3} - 1 \\ 3 \end{pmatrix}$ there must be some evidence of matrix

multiplication seen.

(e)

M1: Identifies and applies an appropriate strategy to find the required angle e.g. cosine rule or scalar product. Note: $AB^2 = 56 - 12\sqrt{3}$ and $AB = 3\sqrt{6} - \sqrt{2} = 5.93\dots$

A1*: Correct value from correct equation

(f)

M1: Uses the given angle with $\frac{1}{2}ab \sin C$ with appropriate a , b and C

Outside spec: uses the cross product $\frac{1}{2}|\overrightarrow{OA} \times \overrightarrow{OB}|$

A1: Correct area to 3 significant figures