Question	Scheme	Marks	AOs
3(a)	$\sum_{r=1}^{n} r^2(r+1) = \sum_{r=1}^{n} r^3 + r^2 = \frac{1}{4}n^2(n+1)^2 + \frac{1}{6}n(n+1)(2n+1)$	M1 A1	1.1b 1.1b
	$= \frac{1}{12}n(n+1)\left[3n(n+1)+2(2n+1)\right]$	dM1	1.1b
	$= \frac{1}{12}n(n+1)\left[3n^2+7n+2\right] = \frac{1}{12}n(n+1)(n+2)(3n+1)$ cso	A1	2.1
		(4)	
(b)	$\sum_{r=k+1}^{3k} r^2(r+1) = \frac{1}{12}(3k)(3k+1)(3k+2)(9k+1) - \frac{1}{12}(k)(k+1)(k+2)(3k+1)$	M1	3.1a
	$= \frac{1}{12}k(3k+1)\left[3(3k+2)(9k+1)-(k+1)(k+2)\right]$ or $= \frac{1}{3}k(3k+1)\left[\frac{3}{4}(3k+2)(9k+1)-\frac{1}{4}(k+1)(k+2)\right]$	M1	1.1b
	$= \frac{1}{12}k(3k+1)\left[80k^2+60k+4\right]$ $= \frac{1}{3}k(3k+1)\left(20k^2+15k+1\right)$ cso	A1	1.1b
		(3)	
(c)	$\frac{25}{3}k(3k+1)\left(20k^2+15k+1\right) = 192k^3(3k+1)$ either $\Rightarrow 25\left(20k^2+15k+1\right) = 576k^2 \Rightarrow 76k^2-375k-25=0$ Or $\Rightarrow 25k\left(20k^2+15k+1\right) = 576k^3 \Rightarrow 76k^3-375k^2-25k=0$ Or $\Rightarrow 25(3k+1)\left(20k^2+15k+1\right) = 576k^2(3k+1) \Rightarrow 228k^3-1049k^2-450k-25=0$ Or $-76k^4+\frac{1049}{3}k^3+150k^2+\frac{25}{3}k=0$	M1	1.1b
	$76k^2-375k-25=0 \Rightarrow k=...$ $76k^3-375k^2-25k=0 \Rightarrow k=...$ $-76k^4+\frac{1049}{3}k^3+150k^2+\frac{25}{3}k=0 \Rightarrow k=...$	M1	1.1b
	$k = 5$ (only)	A1	2.3
		(3)	
			(10 marks)

Notes

(a)

M1: Substitutes at least one of the standard formulae into their expanded expression

A1: Fully correct expression

dM1: Attempts to factorise $\frac{1}{12}n(n+1)$ or $\frac{1}{3}n(n+1)$ having used at least one standard formula correctly at any stage. Dependent on the first M mark.
If they show no method for factorising (use a calculator) they can go from
$3n^3+10n^2+9n+2=(n+1)(n+2)(3n+1)$

A1: Obtains the correct expression or the correct values of a and b, with no errors seen
(b)
M1: Uses the result from part (a) and adopts a correct strategy by attempting
$\sum_{r=1}^{3k} r^2(r+1)-\sum_{r=1}^{k} r^2(r+1)$
M1: Factorises out at least $k(3k+1)$ at any stage, could be done by inspection
A1: Obtains the correct expression with no errors seen.
Note If a candidate does not use part (a) but restarts they can still score marks for the same reasons. If unsure please send to review
(c)
M1: Uses the given equation, substitutes their answer from part (b) and simplifies to either reach

- $Ak^4+Bk^3+Ck^2+Dk\{=0\}$
- $k(Ak^3+Bk^2+Ck+D)\{=0\}$ or $(3k+1)(Ak^3+Bk^2+Ck)\{=0\}$
- $Ak^3+Bk^2+Ck\{=0\}$
- a 3TQ or $k(3k+1)(Ak^2+Bk+C)\{=0\}$

this can be implied by a correct value for k if left unsimplified
M1: Solves their equation as long as solving their $(b)=192k^3(3k+1)$ to find a non zero value for k, including by calculator. You may need to check this.
A1: Selects the appropriate correct answer of $k = 5$. Any other solutions must be clearly rejected.
Note: Correct answer with no working is M0M0A0