Question	Scheme	Marks	AOs
6(a)	Direction: $\pm\left(3\mathbf{i}+4\mathbf{j}-2\mathbf{k}-\left(-2\mathbf{i}-8\mathbf{j}-3\mathbf{k}\right)\right)$	M1	1.1b
	e.g. $\mathbf{r}=3\mathbf{i}+4\mathbf{j}-2\mathbf{k}+\lambda\left(5\mathbf{i}+12\mathbf{j}+\mathbf{k}\right)$ $\mathbf{r}=-2\mathbf{i}-8\mathbf{j}-3\mathbf{k}+\lambda\left(5\mathbf{i}+12\mathbf{j}+\mathbf{k}\right)$	A1	2.5
		(2)	
(b)	$z=0\Rightarrow -2+\lambda=0\Rightarrow \lambda=2\Rightarrow C=...$	M1	1.1b
	$\lambda=2\Rightarrow C$ is $(13,\ 28,\ 0)$	A1	1.1b
		(2)	
(c)	$\left(5\mathbf{i}+12\mathbf{j}+\mathbf{k}\right)\cdot\left(2\mathbf{i}+4\mathbf{j}-2\mathbf{k}\right)=10+48-2$	M1	3.1b
	$56=\sqrt{5^2+12^2+1^2}\sqrt{2^2+4^2+2^2}\cos\theta\Rightarrow\cos\theta=\dfrac{56}{\sqrt{170}\sqrt{24}}$	M1	1.1b
	$\Rightarrow\theta=\text{awrt } 28.8°$	A1	1.1b
		(3)	
(d)	$\mathbf{P}_1-\mathbf{P}_2=3\mathbf{i}+4\mathbf{j}-2\mathbf{k}+\lambda\left(5\mathbf{i}+12\mathbf{j}+\mathbf{k}\right)-\left(\mathbf{i}+3\mathbf{j}-\mathbf{k}+\mu\left(2\mathbf{i}+4\mathbf{j}-2\mathbf{k}\right)\right)$ or $\mathbf{P}_1-\mathbf{P}_2=-2\mathbf{i}-8\mathbf{j}-3\mathbf{k}+\lambda\left(5\mathbf{i}+12\mathbf{j}+\mathbf{k}\right)-\left(\mathbf{i}+3\mathbf{j}-\mathbf{k}+\mu\left(2\mathbf{i}+4\mathbf{j}-2\mathbf{k}\right)\right)$	M1	3.4
	$\left(\left(5\lambda-2\mu+2\right)\mathbf{i}+\left(12\lambda-4\mu+1\right)\mathbf{j}+\left(\lambda+2\mu-1\right)\mathbf{k}\right)\cdot\left(5\mathbf{i}+12\mathbf{j}+\mathbf{k}\right)=0$ $\begin{pmatrix}2+5\lambda-2\mu\\1+12\lambda-4\mu\\-1+\lambda+2\mu\end{pmatrix}\bullet\begin{pmatrix}5\\12\\1\end{pmatrix}$ $\Rightarrow 170\lambda-56\mu=-21$ **AND** $\left(\left(5\lambda-2\mu+2\right)\mathbf{i}+\left(12\lambda-4\mu+1\right)\mathbf{j}+\left(\lambda+2\mu-1\right)\mathbf{k}\right)\cdot\left(2\mathbf{i}+4\mathbf{j}-2\mathbf{k}\right)=0$ $\begin{pmatrix}2+5\lambda-2\mu\\1+12\lambda-4\mu\\-1+\lambda+2\mu\end{pmatrix}\bullet\begin{pmatrix}2\\4\\-2\end{pmatrix}$ $\Rightarrow 56\lambda-24\mu=-10$	M1	3.1b
	$170\lambda-56\mu=-21,\ \ 56\lambda-24\mu=-10\Rightarrow\lambda=...\left(\dfrac{7}{118}\right),\ \mu=...\left(\dfrac{131}{236}\right)$ If using $\mathbf{r}=-2\mathbf{i}-8\mathbf{j}-3\mathbf{k}+\lambda\left(5\mathbf{i}+12\mathbf{j}+\mathbf{k}\right)$ this leads to parameters of $\lambda=...\left(\dfrac{125}{118}\right),\ \mu=...\left(\dfrac{131}{236}\right)$ **Either** $\mathbf{P}_1-\mathbf{P}_2=...\left(\dfrac{70}{59}\mathbf{i}-\dfrac{30}{59}\mathbf{j}+\dfrac{10}{59}\mathbf{k}\right)$ or $\left\|\mathbf{P}_1-\mathbf{P}_2\right\|=...$	M1	3.4

	$\lvert \mathbf{P}_1-\mathbf{P}_2\rvert=\sqrt{\left(\frac{70}{59}\right)^2+\left(\frac{30}{59}\right)^2+\left(\frac{10}{59}\right)^2}=\ldots$	dM1	1.1b
	Awrt 1.3{0} **m** or $\frac{10\sqrt{59}}{59}$ **m units required**	A1	3.2a
		(5)	
	Alternative 1 $\mathbf{P}_1-\mathbf{P}_2=3\mathbf{i}+4\mathbf{j}-2\mathbf{k}+\lambda(5\mathbf{i}+12\mathbf{j}+\mathbf{k})-(\mathbf{i}+3\mathbf{j}-\mathbf{k}+\mu(2\mathbf{i}+4\mathbf{j}-2\mathbf{k}))$	M1	3.4
	Finds $\begin{pmatrix}5\\12\\1\end{pmatrix}\times\begin{pmatrix}2\\4\\-2\end{pmatrix}=\ldots\{-28\mathbf{i}+12\mathbf{j}-4\mathbf{k}\}$ $\begin{pmatrix}2+5\lambda-2\mu\\1+12\lambda-4\mu\\-1+\lambda+2\mu\end{pmatrix}=M\begin{pmatrix}-28\\12\\-4\end{pmatrix}$ Or $\begin{pmatrix}x\\y\\z\end{pmatrix}\bullet\begin{pmatrix}5\\12\\1\end{pmatrix}=5x+12y+z=0$ And $\begin{pmatrix}x\\y\\z\end{pmatrix}\bullet\begin{pmatrix}2\\4\\-2\end{pmatrix}=2x+4y-2z=0$ Leding to a vector e.g. $\begin{pmatrix}7\\-3\\1\end{pmatrix}$	M1	3.1b
	$\begin{matrix}2+5\lambda-2\mu=-28M\\1+12\lambda-4\mu=12M\\-1+\lambda+2\mu=-4M\end{matrix}\Rightarrow\lambda=\ldots\left(\frac{7}{118}\right),\ \mu=\ldots\left(\frac{131}{236}\right),\left\{M=-\frac{5}{118}\right\}$ $\mathbf{P}_1-\mathbf{P}_2=\ldots\left(\frac{70}{59}\mathbf{i}-\frac{30}{59}\mathbf{j}+\frac{10}{59}\mathbf{k}\right)$	M1	3.4
	$\lvert \mathbf{P}_1-\mathbf{P}_2\rvert=\sqrt{\left(\frac{70}{59}\right)^2+\left(\frac{30}{59}\right)^2+\left(\frac{10}{59}\right)^2}$	dM1	1.1b
	Awrt 1.3{0} **m** or $\frac{10\sqrt{59}}{59}$ **m units required**	A1	3.2a
		(5)	
	Alternative 2 outside the spec	M1	3.4

	$\mathbf{r}=\begin{pmatrix}3\\4\\-2\end{pmatrix}+\lambda\begin{pmatrix}5\\12\\1\end{pmatrix}$ and $\mathbf{r}=\begin{pmatrix}1\\3\\-1\end{pmatrix}+\lambda\begin{pmatrix}2\\4\\-2\end{pmatrix}$ $\begin{pmatrix}3\\4\\-2\end{pmatrix}-\begin{pmatrix}1\\3\\-1\end{pmatrix}=\begin{pmatrix}2\\1\\-1\end{pmatrix}$		
	$\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\5&12&1\\2&4&-2\end{vmatrix}=\mathbf{i}(-24-4)-\mathbf{j}(-10-2)+\mathbf{k}(20-24)$ $=-28\mathbf{i}+12\mathbf{j}-4\mathbf{k}$ And $\begin{pmatrix}2\\1\\-1\end{pmatrix}\bullet\begin{pmatrix}-28\\12\\-4\end{pmatrix}=-56+12+4=...\{-40\}$	M1	3.4
	$\lvert-28\mathbf{i}+12\mathbf{j}-4\mathbf{k}\rvert=\sqrt{(-28)^2+12^2+(-4)^2}=...\{\sqrt{944}\}$	dM1	3.1b
	$\lvert\mathbf{P}_1-\mathbf{P}_2\rvert=\left\lvert\frac{-40}{\sqrt{944}}\right\rvert$	M1	1.1b
	Awrt 1.3{0} **m** or $\frac{10\sqrt{59}}{59}$ **m units required**	A1	3.2a
		(5)	
			(12 marks)

Notes

(a)
M1: Subtracts the given coordinates either way round, 2 correct values implies method, and uses as their direction vector.
A1: For a correct equation. Allow any equivalent correct equations but must use the correct notation, starts with **r** = …, can be using column vectors
(b)
M1: Uses $z = 0$ in their P_1 equation to find a value for their parameter and uses this to find the other coordinates.
A1: Deduces the correct coordinates, condone written as a vector.
(c)
M1: Realises the scalar product between their direction from part (a) and the direction vector extracted from the P_2 equation is required and calculates its value
M1: Completes the method and at least reaches a value for cosine of the angle. Must be attempting to use the direction vectors
A1: Correct acute angle
(d)
M1: Uses the model to form a general vector connecting the two pipes, condone sign slips if the intention is clear. Condone use of the same parameter for this mark. They will be unable to score any more marks
M1: Recognises that the scalar product between the general vector **and** the directions of the lines = 0 and uses this to form 2 simultaneous equations in terms of their parameters. Follow through on their line from part (a)

M1: Attempts to solve the simultaneous equations, writing a value for each parameter is sufficient.
Then uses the values of their parameters in the model and **either**

- shows the shortest vector connecting the 2 lines
- proceeds to a distance with no incorrect working seen.

dM1: Uses their values of their parameters to find the vector which if not correct must be stated and then finds the modulus of their shortest vector to find the shortest distance. Maybe implied by a correct answer. Dependent on previous method mark.
A1: Awrt 1.3(0) **m**

Alternative 1

M1: Uses the model to form a general vector connecting the two pipes, condone sign slips if the intention is clear.
M1: Finds the normal vector to the direction vectors by any method and sets the general equation of the line equal to a multiple of the normal vector.
M1: Attempts to solve the simultaneous equations, writing a value for each parameter is sufficient.
Then uses the values of their parameters in the model and **either**

- shows the shortest vector connecting the 2 lines
- proceeds to a distance with no incorrect working seen.

dM1: Uses their values of their parameters to find the vector which if not correct must be stated and then finds the modulus of their shortest vector to find the shortest distance. Maybe implied by a correct answer. Dependent on previous method mark.
A1: Awrt 1.3(0) **m**

Alternative 2 outside spec

M1: Using their equations to find the vector between the coordinates $(\mathbf{a}-\mathbf{c})$
M1: Find the cross product of their directions $(\mathbf{b}\times\mathbf{d})$ and finds the dot product of their $(\mathbf{a}-\mathbf{c})\bullet(\mathbf{b}\times\mathbf{d})$
M1: Finds their $|\mathbf{b}\times\mathbf{d}|$
M1: Uses $\left|\dfrac{(\mathbf{a}-\mathbf{c})\bullet(\mathbf{b}\times\mathbf{d})}{|\mathbf{b}\times\mathbf{d}|}\right|$
A1: Awrt 1.3(0) **m.**