| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 7(i) | $n=1,\quad lhs=\frac{1}{1(2)}=\frac{1}{2}\quad rhs=\frac{1}{1+1}=\frac{1}{2}$<br>So the result is true for $n = 1$ | B1 | 2.2a |
| | Assume true for $n = k$ so $\sum_{r=1}^{k}\frac{1}{r(r+1)}=\frac{k}{k+1}$ | M1 | 2.4 |
| | $\sum_{r=1}^{k+1}\frac{1}{r(r+1)}=\frac{k}{k+1}+\frac{1}{(k+1)(k+2)}=\frac{k(k+2)}{(k+1)(k+2)}+\frac{1}{(k+1)(k+2)}$ | M1 | 2.1 |
| | $=\frac{k(k+2)+1}{(k+1)(k+2)}=\frac{k^2+2k+1}{(k+1)(k+2)}=\frac{(k+1)^2}{(k+1)(k+2)}=\frac{\underline{k+1}}{\underline{k+1}+1}$ cso | A1 | 1.1b |
| | **If true for *n* = *k* then** it has been shown **true for *n* = *k* + 1** and as it is **true for *n* = 1,** the statement is **true for all positive integers *n*.** | A1 | 2.4 |
| | | **(5)** | |
| 7(ii) | **Way 1: f (*k* + 1)**<br>$n=1,\quad 3^{2n+4}-2^{2n}=3^6-2^2=729-4=725$<br>So the result is true for $n = 1$ as 725 is divisible by 5 | B1 | 2.2a |
| | Assume true for $n = k$ so is $3^{2k+4}-2^{2k}$ divisible by 5 | M1 | 2.4 |
| | $\mathrm{f}(k+1)=3^{2(k+1)+4}-2^{2(k+1)}=3^{2k+6}-2^{2k+2}$<br>Look for<br>$A\times3^{2k+4}-A\times2^{2k}+B\times2^{2k}$ or $A\times3^{2k+4}-A\times2^{2k}+B\times3^{2k+4}$<br>$9\times3^{2k+4}-9\times2^{2k}+5\times2^{2k}$ or $4\times3^{2k+4}-4\times2^{2k}+5\times3^{2k+4}$ | M1 | 2.1 |
| | $=9\mathrm{f}(k)+5\times2^{2k}$ or $=4\mathrm{f}(k)+5\times3^{2k+4}$ | A1 | 1.1b |
| | **If true for *n* = *k* then** it has been shown **true for *n* = *k* + 1** and as it is **true for *n* = 1,** the statement is **true for all positive integers *n*.** | A1 | 2.4 |
| | | **(5)** | |
| | **Way 2: f (*k* + 1)- f (*k* )**<br>$n=1,\quad 3^{2n+4}-2^{2n}=3^6-2^2=729-4=725$<br>So the result is true for $n = 1$ as 725 is divisible by 5 | B1 | 2.2a |
| | Assume true for $n = k$ so $3^{2k+4}-2^{2k}$ is divisible by 5 | M1 | 2.4 |
| | $\mathrm{f}(k+1)-\mathrm{f}(k)=3^{2(k+1)+4}-2^{2(k+1)}-3^{2k+4}+2^{2k}$<br>Look for<br>$A\times3^{2k+4}-A\times2^{2k}+B\times2^{2k}$ or $A\times3^{2k+4}-A\times2^{2k}+B\times3^{2k+4}$<br>$=8\times3^{2k+4}-8\times2^{2k}+5\times2^{2k}$ or $3\times3^{2k+4}-3\times2^{2k}+5\times3^{2k+4}$ | M1 | 2.1 |
| | $\mathrm{f}(k+1)=9\mathrm{f}(k)+5\times2^{2k}$ | A1 | 1.1b |

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| | or<br>$\mathrm{f}\left(k+1\right)=4\mathrm{f}\left(k\right)+5\times 3^{2k+4}$ | | |
| | **If true for *n* = *k* then** it has been shown **true for *n* = *k* + 1** and as it is **true for *n* = 1,** the statement is **true for all positive integers *n*.** | A1 | 2.4 |
| | | **(5)** | |
| | **Way 3: $\mathbf{f}(\boldsymbol{k})= 5\boldsymbol{M}$**<br>$n=1,\quad 3^{2n+4}-2^{2n}=3^6-2^2=729-4=725$<br>So the result is true for *n* = 1 as 725 is divisible by 5 | B1 | 2.2a |
| | Assume true for *n* = *k* so $3^{2k+4}-2^{2k}=5M$ is divisible by 5 | M1 | 2.4 |
| | $\mathrm{f}\left(k+1\right)=3^{2(k+1)+4}-2^{2(k+1)}\left(=3^{2k+6}-2^{2k+2}\right)$<br>$\mathrm{f}\left(k+1\right)=3^2\times 3^{2k+4}-2^2\times 2^{2k}=3^2\times\left(5M+2^{2k}\right)-2^2\times 2^{2k}$<br>OR<br>$\mathrm{f}\left(k+1\right)=3^2\times 3^{2k+4}-2^2\times 2^{2k}=3^2\times 3^{2k+4}-2^2\times\left(3^{2k+4}-5M\right)$ | M1 | 2.1 |
| | $\mathrm{f}(k+1)= 45M + 5\times 2^{2k}$ OR $\mathrm{f}(k+1)= 5\times 3^{2k+4} + 20M$ o.e. | A1 | 1.1b |
| | **If true for *n* = *k* then** it has been shown **true for *n* = *k* + 1** and as it is **true for *n* = 1,** the statement is **true for all positive integers *n*.** | A1 | 2.4 |
| | | **(5)** | |
| | **Way 4: $\mathbf{f}(\boldsymbol{k}+1)- \boldsymbol{m}\,\mathbf{f}(\boldsymbol{k})$**<br>$n=1,\quad 3^{2n+4}-2^{2n}=3^6-2^2=729-4=725$<br>So the result is true for *n* = 1 as 725 is divisible by 5 | B1 | 2.2a |
| | Assume true for *n* = *k* so $3^{2k+4}-2^{2k}$ is divisible by 5 | M1 | 2.4 |
| | $\mathrm{f}(k+1)- m\mathrm{f}(k)= 3^{2k+6}- 2^{2k+2}- m\left(3^{2k+4}- 2^{2k}\right)$<br>$3^2\times 3^{2k+4}- 2^2\times 2^{2k}- m\times 3^{2k+4}+ m\times 2^{2k}$<br>$(9- m)\times 3^{2k+4}- 4\times 2^{2k}+ m\times 2^{2k}$<br>$(9- m)\times\left(3^{2k+4}- 2^{2k}\right)+ 5\times 2^{2k}$ | M1 | 2.1 |
| | $\mathrm{f}(k+1)= (9- m)\times \mathrm{f}(k)+ 5\times 2^{2k}+ m\mathrm{f}(k)$<br>$\mathrm{f}(k+1)= (9- m)\times\left(3^{2k+4}- 2^{2k}\right)+ 5\times 2^{2k}+ m\mathrm{f}(k)$<br>Note if $m=4$ leads to $5\times\left(3^{2k+4}\right)$ | A1 | 1.1b |
| | **If true for *n* = *k* then** it has been shown **true for *n* = *k* + 1** and as it is **true for *n* = 1,** the statement is **true for all positive integers *n*.** | A1 | 2.4 |
| | | **(5)** | |
| | | | **(10 marks)** |
| | **Notes** | | |
| **(i) Must be using induction to score any marks** | | | |

B1: Shows that the result holds for $n = 1$. Minimum LHS = $\frac{1}{1(2)}$ RHS $= \frac{1}{1+1}$

M1: Makes a statement that assumes the result is true for some value of $n$, say $k$
M1: Attempts to add the next term and makes progress by attempting a common denominator.
A1: Achieves a correct expression in terms of $k + 1$, with no errors seen cso. Alternatively finds $\sum_{r=1}^{k+1} \frac{1}{r(r+1)} = \frac{k+1}{k+2}$ and reaches the same expression using induction.
A1: Correct conclusion. This mark is dependent on all previous marks. It is gained by conveying the ideas of **all four bold** points either at the end of their solution or as a narrative in their solution. **Not dependent on B1 as long as attempted**

(ii)

**Way 1: f (*k* + 1)**

B1: Shows that the result holds for $n = 1$
M1: Makes a statement that assumes the result is true for some value of $n$, say $k$
M1: Attempts f $(k + 1)$ and attempts to express in terms of f $(k)$
A1: Achieves a correct expression in terms of f $(k)$
A1: Correct conclusion. This mark is dependent on all previous marks. It is gained by conveying the ideas of **all four bold** points either at the end of their solution or as a narrative in their solution. **Not dependent on B1 as long as attempted**

**Way 2: f (*k* + 1)- f (*k*)**

B1: Shows that the result holds for $n = 1$
M1: Makes a statement that assumes the result is true for some value of $n$, say $k$
M1: Attempts f $(k + 1)$ – f $(k)$ and attempts to express in terms of f $(k)$
A1: Achieves a correct expression for f $(k + 1)$ in terms of f $(k)$
A1: Correct conclusion. This mark is dependent on all previous marks. It is gained by conveying the ideas of **all four bold** points either at the end of their solution or as a narrative in their solution. **Not dependent on B1 as long as attempted**

**Way 3: f (*k*)= 5*M***

B1: Shows that the result holds for $n = 1$
M1: Makes a statement that assumes the result is true for some value of $n$, say $k$
M1: Attempts f$(k + 1)$ and writes in terms of $5M$.
A1: Achieves a correct expression for f $(k + 1)$ in terms of $M$ and $2^{2k}$ or $M$ and $3^{2k+4}$
A1: Correct conclusion. This mark is dependent on all previous marks. It is gained by conveying the ideas of **all four bold** points either at the end of their solution or as a narrative in their solution. **Not dependent on B1 as long as attempted**

**Way 4: f (*k* + 1)- *m*f (*k*)**

B1: Shows that the result holds for $n = 1$
M1: Makes a statement that assumes the result is true for some value of $n$, say $k$
M1: Attempts f $(k + 1)$ – m f $(k)$ and attempts to express in terms of f $(k)$
A1: Achieves a correct expression for f $(k + 1)$ in terms of f $(k)$
A1: Correct conclusion. This mark is dependent on all previous marks. It is gained by conveying the ideas of **all four bold** points either at the end of their solution or as a narrative in their solution. **Not dependent on B1 as long as attempted**

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| **Note conclusion may be in terms of** $\mathrm{f}(1), \mathrm{f}(k), \mathrm{f}(k+1)$ |