4.

$$\mathbf{A} = \begin{pmatrix} -1 & -2 & -7 \\ 3 & k & 2 \\ 1 & 1 & 4 \end{pmatrix} \qquad \mathbf{B} = \begin{pmatrix} 4k-2 & 1 & 7k-4 \\ -10 & 3 & -19 \\ 3-k & -1 & 6-k \end{pmatrix}$$

where k is a constant.

(a) Determine the value of the constant c for which

$$\mathbf{AB} = (3k + c)\mathbf{I}$$

(2)

(b) Hence determine the value of k for which $\mathbf{A}^{-1}$ does not exist.

(2)

Given that $\mathbf{A}^{-1}$ does exist,

(c) write down $\mathbf{A}^{-1}$ in terms of k.

(1)

(d) Use the answer to part (c) to solve the simultaneous equations

$$\begin{aligned} -x - 2y - 7z &= 10 \\ 3x + ky + 2z &= 3 \\ x + y + 4z &= 1 \end{aligned}$$

giving the values of x, y and z in simplest form in terms of k.

(3)