8.

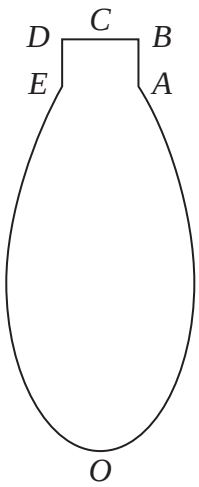

Figure 1

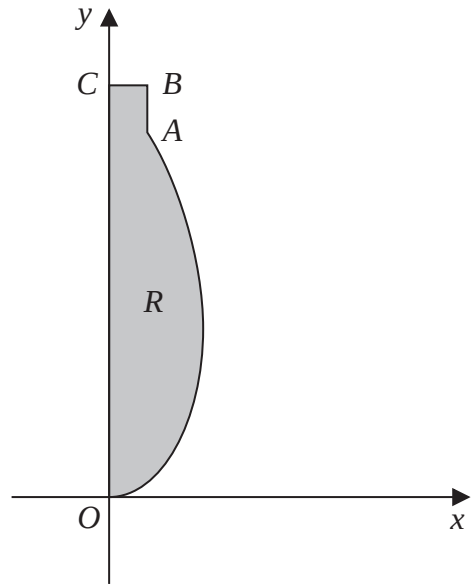

Figure 2

Figure 1 shows the central vertical cross-section, *OABCDEO*, of the design for a solid glass ornament.

Figure 2 shows the finite region, *R*, which is bounded by the *y*-axis, the horizontal line *CB*, the vertical line *BA*, and the curve *AO*.

The ornament is formed by rotating the region *R* through 360° about the *y*-axis.

The curve *AO* is modelled by the equation

$$x = ky^2 + \sqrt{y} \qquad 0 \leqslant y \leqslant 4$$

where *k* is a constant.

The point *A* has coordinates (0.4, 4) and the point *B* has coordinates (0.4, 4.5)

The units are centimetres.

(a) Determine the value of *k* according to this model.

(2)

(b) Use algebraic integration to determine the exact volume of glass that would be required to make the ornament, according to the model.

(7)

(c) State a limitation of the model.

(1)

When the ornament was manufactured, $9\,\text{cm}^3$ of glass was required.

(d) Use this information and your answer to part (b) to evaluate the model, explaining your reasoning.

(1)