Question	Scheme		Marks	AOs
4 (i)	$\begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} \Rightarrow \begin{matrix} y = \\ y = \end{matrix}$	$ = -x \\ = -x \\ = -x $	B1	1.1b
			(1)	
(ii) (a)	$ \begin{pmatrix} 3 & -1 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} x \\ mx + c \end{pmatrix} = \begin{pmatrix} X \\ mX + c \end{pmatrix} $ leading to an equation in <i>x</i> , <i>m</i> , <i>c</i> and <i>X</i>		M1	3.1a
	3x - (mx + c) = X and $3x + 4(mx + c) = mX + c$		A1	1.1b
	3x + 4(mx + c) = m(3x - (mx + c)) + c (3 + 4m = 3m - m ²) (4c = -mc + c) leading to m ² + m + 3 = 0 $\Rightarrow b^2 - 4ac = (1)^2 - 4(1)(3) =$		dM1	2.1
	Alternatively, solves $4c = -mc + c =$	<i>⇒ m =</i>		
	Correct expression for the discriminant = $\{-11\} < 0$ therefore there are no invariant lines.	m = -3 and shows a contradiction in $3 + 4m = 3m - m^2$ therefore there are no invariant lines.	A1	2.4
	$\frac{\text{Alternative}}{\begin{pmatrix}3 & -1\\3 & 4\end{pmatrix}\begin{pmatrix}x\\mx\end{pmatrix} = \begin{pmatrix}X\\mX\end{pmatrix} \text{leading to an equation in } x, m \text{ and } X}$ $3x - (mx) = X \text{ and } 3x + 4(mx) = mX$ $3x + 4(mx) = m(3x - (mx))$ $\text{leading to } 3 + 4m = 3m - m^2$ $m^2 + m + 3 = 0 \Rightarrow b^2 - 4ac = (1)^2 - 4(1)(3) = \dots$ $\text{Correct expression for the discriminant } = \{-11\} < 0 \text{ therefore there are no invariant lines that pass through the origin so no invariant lines.}$		(4)	
			M1	3.1a
			A1	1.1b
			dM1	2.1
			A1	2.4
			(4)	
(ii) (b)	det $\mathbf{M} = 15$ Area of original triangle = $\left \frac{1}{2}(8-k)2\right $ $ 8-k = 75 \div ``15'' \Rightarrow k =$		B1	1.1b
			M1	3.1a
	k = 3 or $k = 13$		A1	1.1b

	k = 3 and $k = 13$	A1	1.1b
		(4)	
(9 marks)			

Notes

(i)

B1: Correct line

(ii) (a)

M1: Sets up a matrix equation in an attempt to find a fixed line and extracts at least one equation.

A1: Correct equations.

dM1: Eliminates X from the simultaneous equations and equates the coefficients of x leading to a quadratic equation in terms of m, then finds the value of the discriminant, which can be seen in an attempt to solve the quadratic using the formula.

Alternatively solves 4c = -mc + c and finds a value for *m*

Note: If the quadratic equation in *m* is solved on a calculator and complex roots given this is M0 as they are not showing why there are no real roots.

A1: Correct expression for the discriminant, states < 0 and draws the required conclusion. Alternatively, correct value for *m*, shows a contradiction in $3 + 4m = 3m - m^2$ and draws the required conclusion.

Alternative

M1: Sets up a matrix equation in an attempt to find a fixed line and extracts at least one equation.

A1: Correct equations.

dM1: Eliminates X from the simultaneous equations and equates the coefficients of x leading to a quadratic equation in terms of m. then finds the value of the discriminant which can be seen in an attempt to solve the quadratic using the formula.

A1: Correct expression for the discriminant, states < 0 and draws the required conclusion.

(ii) (b)

B1: det M = 15 oe eg det M = 12 - (-3) or det M = 3 (4) - (-1)(3)

M1: Finds an expression in terms of k for the area of the original triangle and equates this to

 $\frac{75}{\text{"their determinant"}}$, proceeding to find at least one value for k

A1: k = 3 or k = 13

A1: Both values correct