

$$f(x) = 4x^3 - 12x^2 + 2x - 6$$

(a) Use the factor theorem to show that $(x - 3)$ is a factor of $f(x)$.

(2)

(b) Hence show that 3 is the only real root of the equation $f(x) = 0$

(4)

$$\begin{aligned} \text{(a)} \quad f(3) &= 4(3)^3 - 12(3)^2 + 2(3) - 6 \\ &= 108 - 108 + 6 - 6 \\ &= 0 \end{aligned}$$

so, $(x-3)$ is a factor (2 marks)

$$\begin{array}{r} \text{(b)} \quad x-3 \overline{) \begin{array}{r} 4x^2 + 2 \\ 4x^3 - 12x^2 + 2x - 6 \\ \underline{-(4x^3 - 12x^2)} \\ 0 + 2x - 6 \\ \underline{-(2x - 6)} \\ 0 \end{array}} \end{array}$$

so,

$$f(x) = (x-3)(4x^2 + 2) \quad (2 \text{ marks})$$

$$\begin{aligned} \text{discriminant of } 4x^2 + 2 &= (0)^2 - 4(4)(2) \\ &= -32 < 0 \end{aligned}$$

so, $4x^2 + 2 = 0$ has no real solutions

$$\begin{aligned} \text{so } (x-3) = 0 \text{ is only real root of } f(x) = 0 \\ \Rightarrow x = 3 \end{aligned}$$

(2 marks)