

6. Prove, from first principles, that the derivative of $3x^2$ is $6x$.

(4)

$$\begin{aligned}\frac{dy}{dx} &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{3(x+h)^2 - 3x^2}{h} \quad (1 \text{ mark})\end{aligned}$$

$$= \lim_{h \rightarrow 0} \frac{3x^2 + 6hx + 3h^2 - 3x^2}{h} \quad (1 \text{ mark})$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{3x^2} + 6hx + 3h^2 - \cancel{3x^2}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{6x + 3h}{1} \quad (1 \text{ mark})$$

as $h \rightarrow 0$, ~~$3h$~~ $\rightarrow 3h \rightarrow 0$

$$\text{so } \frac{dy}{dx} = 6x \quad (1 \text{ mark})$$