

11. (a) Prove that for all positive values of x and y

$$\sqrt{xy} \leq \frac{x+y}{2} \quad (2)$$

(b) Prove by counter example that this is not true when x and y are both negative. (1)

(a) since x and y are positive,
their square roots are real, so,

$$(\sqrt{x} - \sqrt{y})^2 \geq 0 = x - 2\sqrt{x}\sqrt{y} + y \geq 0 \quad (1 \text{ mark})$$

which rearranges to $\sqrt{xy} \leq \frac{x+y}{2}$ (1 mark)

(b) Consider $x = -3$, $y = -5$

$$\text{then } \sqrt{xy} = \sqrt{15}$$

$$\frac{x+y}{2} = -4$$

$$\sqrt{15} \not\leq -4$$

so result does not hold for all negative x, y

(1 mark)