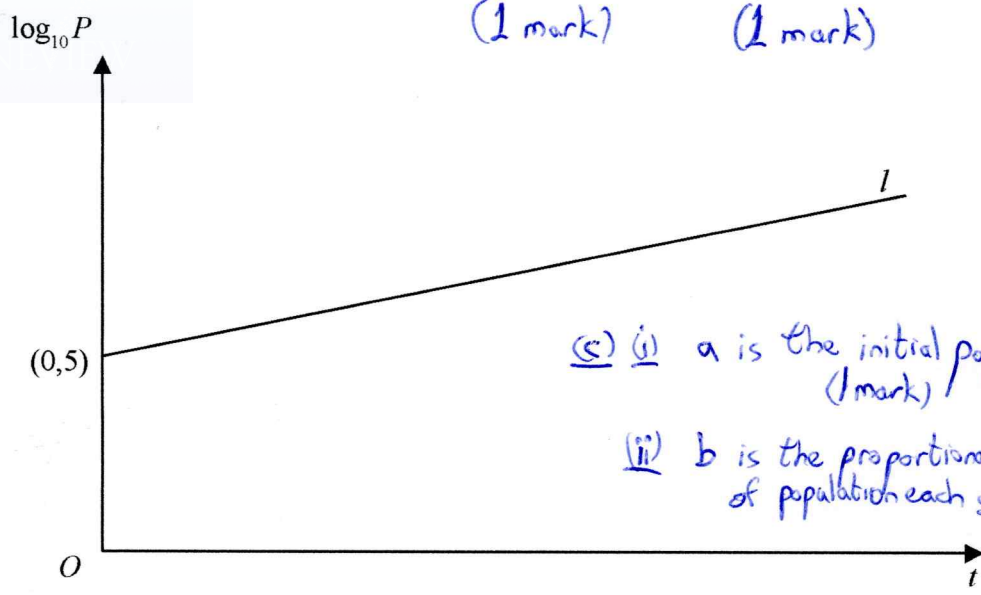


14.

(a)  $\log_{10} P = mt + c = \frac{1}{200}t + c$   
 (1 mark) (1 mark)



(c) (i) a is the initial population (1 mark)  
 (ii) b is the proportional increase of population each year (1 mark)

Figure 2

A town's population,  $P$ , is modelled by the equation  $P = ab^t$ , where  $a$  and  $b$  are constants and  $t$  is the number of years since the population was first recorded.

The line  $l$  shown in Figure 2 illustrates the linear relationship between  $t$  and  $\log_{10} P$  for the population over a period of 100 years.

The line  $l$  meets the vertical axis at  $(0, 5)$  as shown. The gradient of  $l$  is  $\frac{1}{200}$ .

(a) Write down an equation for  $l$ .

(b)  $\log_{10} P = \log_{10} a + t \log_{10} b$  (2)  
 $= c + mt$  (1 mark)  
 (4)

(b) Find the value of  $a$  and the value of  $b$ .

(c) With reference to the model interpret

y-intercept is 5 gradient is  $\frac{1}{200}$   
 so  $5 = \log_{10} a$  so  $\frac{1}{200} = \log_{10} b$   
 $a = 10^5$   $b = 10^{\frac{1}{200}}$   
 (1 mark) OR (1 mark) (2)

- (i) the value of the constant  $a$ ,
- (ii) the value of the constant  $b$ .

(d) Find

$\Rightarrow a = 100,000$  (1 mark)  $b = 1.011579...$   
 $= 1.0116$  5sf (1 mark)

- (i) the population predicted by the model when  $t = 100$ , giving your answer to the nearest hundred thousand, (2) (1)  $\log_{10} P = 5 + \frac{1}{200}(100) = 5.5 \Rightarrow P = 10^{5.5} = 300,000$  (to nearest 1000) (1 mark)
- (ii) the number of years it takes the population to reach 200 000, according to the model.

$200,000 = 10^5 (10^{\frac{1}{200}})^t \Rightarrow \log_{10} 200,000 = \frac{1}{200}t + 5 \Rightarrow t = 60.2$  years 3sf (2 marks) (2)

(e) State two reasons why this may not be a realistic population model.

- 100 years is a long time and wars or disease may occur
  - inaccuracies in measuring gradient could make a big difference
  - population growth may not be proportional to population size
  - the model predicts unlimited growth
- any 2 for (2 marks)