

15.

Diagram not
drawn to scale

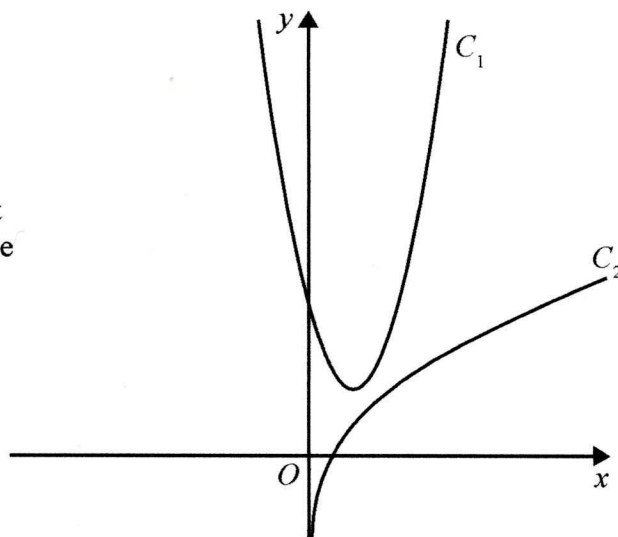


Figure 3

The curve C_1 , shown in Figure 3, has equation $y = 4x^2 - 6x + 4$.

The point $P\left(\frac{1}{2}, 2\right)$ lies on C_1 .

The curve C_2 , also shown in Figure 3, has equation $y = \frac{1}{2}x + \ln(2x)$.

The normal to C_1 at the point P meets C_2 at the point Q .

Find the exact coordinates of Q .

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(8)

$$\frac{dy}{dx} = 8x - 6 \quad (1 \text{ mark})$$

$$\text{gradient at } P = 8\left(\frac{1}{2}\right) - 6 = -2 \quad (1 \text{ mark})$$

$$\text{normal gradient at } P = -\frac{1}{m} = -\frac{1}{-2} = \frac{1}{2} \quad (1 \text{ mark})$$

equation of normal at P is

$$\frac{y-2}{x-\frac{1}{2}} = \frac{1}{2} \Rightarrow y = \frac{1}{2}x + \frac{7}{4} \quad (1 \text{ mark})$$

when normal meets C_2 at Q ,

$$\frac{1}{2}x + \ln(2x) = \frac{1}{2}x + \frac{7}{4}$$

$$\ln(2x) = \frac{7}{4}$$

$$x = \frac{1}{2}e^{\frac{7}{4}}$$

(2 marks)

$$y = \frac{1}{2}\left(\frac{1}{2}e^{\frac{7}{4}}\right) + \frac{7}{4}$$

$$= \frac{1}{4}e^{\frac{7}{4}} + \frac{7}{4}$$

, so Q is $\left(\frac{1}{2}e^{\frac{7}{4}}, \frac{1}{4}e^{\frac{7}{4}} + \frac{7}{4}\right)$ (2 marks)