

16.

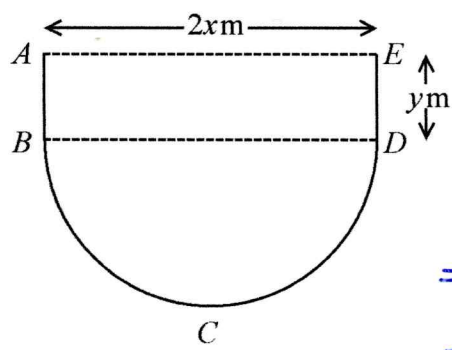


Figure 4

Figure 4 shows the plan view of the design for a swimming pool.

The shape of this pool $ABCDEA$ consists of a rectangular section $ABDE$ joined to a semicircular section BCD as shown in Figure 4.

Given that $AE = 2x$ metres, $ED = y$ metres and the area of the pool is 250m^2 ,

(a) show that the perimeter, P metres, of the pool is given by

$$P = 2x + \frac{250}{x} + \frac{\pi x}{2}$$

(c) $P = 2x + \frac{250}{x} + \frac{\pi x}{2}$
 $\frac{dP}{dx} = 2 - \frac{250}{x^2} + \frac{\pi}{2}$ (2 marks)
 $\frac{dP}{dx} = 0$
 $\Rightarrow \frac{250}{x^2} = 2 + \frac{\pi}{2}$
 $\Rightarrow x = +\sqrt{\frac{250}{2 + \frac{\pi}{2}}}$
 $= 8.367\dots$ (1 mark)

(b) Explain why $0 < x < \sqrt{\frac{500}{\pi}}$

(c) $P = 2(8.367\dots) + \frac{250}{8.367\dots} + \frac{\pi(8.367\dots)}{2}$
 $= 59.756\dots$ (4)
 $= 59.8\text{m}$ 3sf (1 mark) (2)

(c) Find the minimum perimeter of the pool, giving your answer to 3 significant figures.

(a) Area = $2xy + \frac{1}{2}(\pi x^2)$
 (radius = $\frac{2x}{2} = x$)
 $= 2xy + \frac{1}{2}\pi x^2$ (1 mark)
 $= 250$, given

Rearrange to eliminate y
 $\Rightarrow y = \frac{250 - \frac{1}{2}\pi x^2}{2x}$ (1 mark)

$P = 2x + 2y + \frac{1}{2}(2\pi x)$
 $= 2x + 2y + \pi x$
 subst. for y gives
 $P = 2x + 2\left(\frac{250 - \frac{1}{2}\pi x^2}{2x}\right) + \pi x$ (1 mark)

$P = 2x + \frac{500}{2x} - \frac{\pi x^2}{2x} + \pi x$
 $= 2x + \frac{250}{x} + \frac{\pi x}{2}$ (1 mark)

(b) as distances, $x \ \& \ y > 0$
 $x > 0, y = \frac{250 - \frac{1}{2}\pi x^2}{2x} > 0$
 $x > 0, \text{ so } 250 - \frac{1}{2}\pi x^2 > 0$
 $\frac{1}{2}\pi x^2 < 250$
 $x^2 < \frac{500}{\pi}$
 $0 < x < \sqrt{\frac{500}{\pi}}$ (2 marks)