

17. A circle C with centre at $(-2, 6)$ passes through the point $(10, 11)$.

(a) Show that the circle C also passes through the point $(10, 1)$.

(3)

The tangent to the circle C at the point $(10, 11)$ meets the y axis at the point P and the tangent to the circle C at the point $(10, 1)$ meets the y axis at the point Q .

(b) Show that the distance PQ is 58 explaining your method clearly.

(7)

(a) Circle equation is

$$(x - (-2))^2 + (y - 6)^2 = r^2$$

$$(x + 2)^2 + (y - 6)^2 = r^2$$

at $(10, 11)$

$$(10 + 2)^2 + (11 - 6)^2 = r^2$$

$$144 + 25 = r^2$$

$$r = 13 \text{ and}$$

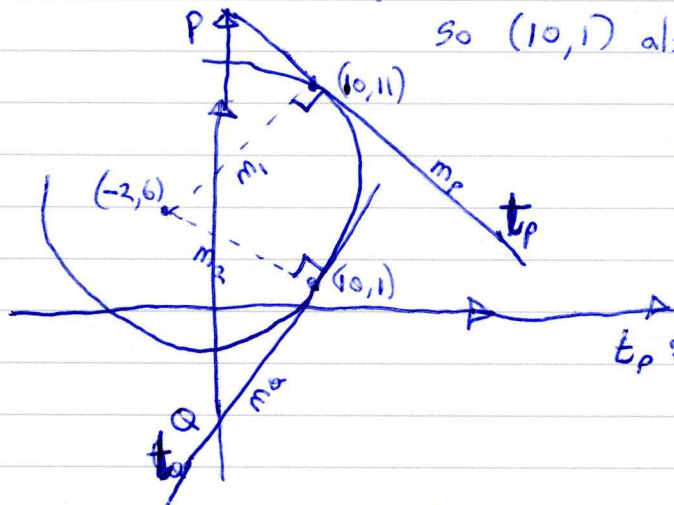
$$\text{Circle is } (x + 2)^2 + (y - 6)^2 = 13^2 \quad (1 \text{ mark})$$

at $(10, 1)$,

$$(10 + 2)^2 + (1 - 6)^2 = 144 + 25 = 13^2$$

So $(10, 1)$ also lies on circle. (2 marks)

(b)



$$m_1 = \frac{11 - 6}{10 - (-2)} = \frac{5}{12} \quad (1 \text{ mark})$$

$$m_p = -\frac{1}{\frac{5}{12}} = -\frac{12}{5} \quad (1 \text{ mark})$$

$$t_p: \frac{y - 11}{x - 10} = -\frac{12}{5}$$

$$\Rightarrow y = -\frac{12}{5}(x - 10) + 11 \quad (1 \text{ mark})$$

$$\Rightarrow y = -\frac{12}{5}x + 35$$

~~$$y = -\frac{12}{5}x + 35$$~~

so P is $(0, 35)$ (1 mark)

$$m_2 = \frac{1 - 6}{10 - (-2)} = -\frac{5}{12}, \quad m_q = -\frac{1}{-\frac{5}{12}} = \frac{12}{5} \quad (1 \text{ mark})$$

$$t_q: \frac{y - 1}{x - 10} = \frac{12}{5} \Rightarrow y = \frac{12}{5}x - 23$$

so Q is $(0, -23)$ (1 mark)

$$\text{Distance } PQ = 35 - (-23) = 58 \quad (1 \text{ mark})$$