

Question	Scheme	Marks	AOs
14 (a)	Attempts to complete the square $(x \pm 3)^2 + (y \pm 5)^2 = \dots$	M1	1.1b

	(i) Centre $(3, -5)$	A1	1.1b
	(ii) Radius 5	A1	1.1b
		(3)	
(b)	Uses a sketch or otherwise to deduce $k = 0$ is a critical value	B1	2.2a
	Substitute $y = kx$ in $x^2 + y^2 - 6x + 10y + 9 = 0$	M1	3.1a
	Collects terms to form correct 3TQ $(1+k^2)x^2 + (10k-6)x + 9 = 0$	A1	1.1b
	Attempts $b^2 - 4ac \dots 0$ for their a, b and c leading to values for k $"(10k-6)^2 - 36(1+k^2) \dots 0" \rightarrow k = \dots, \dots$ $\left(0 \text{ and } \frac{15}{8}\right)$	M1	1.1b
	Uses $b^2 - 4ac > 0$ and chooses the outside region (see note) for their critical values (Both a and b must have been expressions in k)	dM1	3.1a
	Deduces $k < 0, k > \frac{15}{8}$ oe	A1	2.2a
		(6)	

(9 marks)

Notes

(a)

M1: Attempts $(x \pm 3)^2 + (y \pm 5)^2 = \dots$

This mark may be implied by candidates writing down a centre of $(\pm 3, \pm 5)$ or $r^2 = 25$

(i) **A1:** Centre $(3, -5)$

(ii) **A1:** Radius 5. Do not accept $\sqrt{25}$

Answers only (no working) scores all three marks

(b)

B1: Uses a sketch or their subsequent quadratic to deduce that $k = 0$ is a critical value.

You may award for the correct $k < 0$ but award if $k \leq 0$ or even with greater than symbols

M1: Substitutes $y = kx$ in $x^2 + y^2 - 6x + 10y + 9 = 0$ or their $(x \pm 3)^2 + (y \pm 5)^2 = \dots$ to form an

equation in just x and k . It is possible to substitute $x = \frac{y}{k}$ into their circle equation to form an equation in just y and k .

A1: Correct 3TQ $(1+k^2)x^2 + (10k-6)x + 9 = 0$ with the terms in x collected. The " $= 0$ " can be implied by subsequent work. This may be awarded from an equation such as

$x^2 + k^2x^2 + (10k-6)x + 9 = 0$ so long as the correct values of a, b and c are used in $b^2 - 4ac \dots 0$.

FYI The equation in y and k is $(1+k^2)y^2 + (10k^2 - 6k)y + 9k^2 = 0$ oe

M1: Attempts to find two critical values for k using $b^2 - 4ac \dots 0$ or $b^2 \dots 4ac$ where \dots could be " $=$ " or any inequality.

dM1: Finds the outside region using their critical values. Allow the boundary to be included. It is dependent upon all previous M marks and both a and b must have been expressions in k .

Note that it is possible that the correct region could be the inside region if the coefficient of k^2 in $4ac$ is larger than the coefficient of k^2 in b^2 Eg.

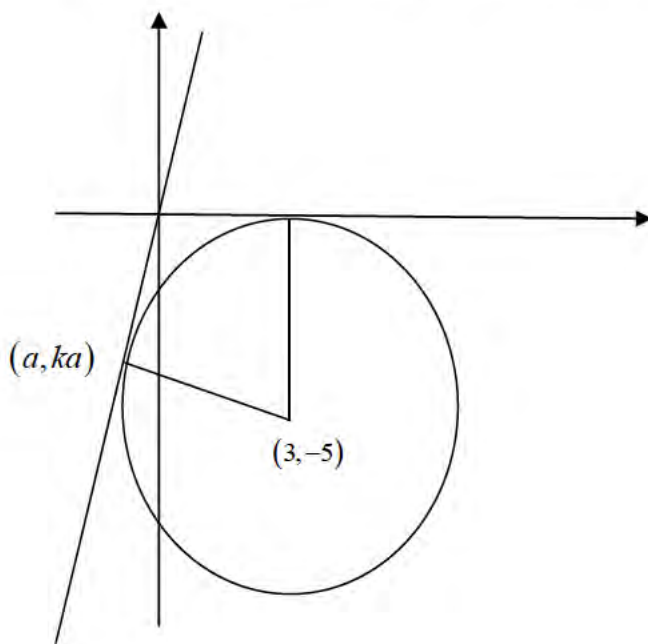
$b^2 - 4ac = (k-6)^2 - 4 \times (1+k^2) \times 9 > 0 \Rightarrow -35k^2 - 12k > 0 \Rightarrow k(35k+12) < 0$

A1: Deduces $k < 0, k > \frac{15}{8}$. This must be in terms of k .

Allow exact equivalents such as $k < 0 \cup k > 1.875$

but not allow $0 > k > \frac{15}{8}$ or the above with AND, & or \cap between the two inequalities

Alternative using a geometric approach with a triangle with vertices at $(0, 0)$, and $(3, -5)$



Alt (b)	Uses a sketch or otherwise to deduce $k = 0$ is a critical value	B1	2.2a
	Distance from (a, ka) to $(0, 0)$ is $3 \Rightarrow a^2(1+k^2) = 9$	M1	3.1a
	Tangent and radius are perpendicular $\Rightarrow k \times \frac{ka+5}{a-3} = -1 \Rightarrow a(1+k^2) = 3-5k$	M1	3.1a
	Solve simultaneously, (dependent upon both M's)	dM1	1.1b
	$k = \frac{15}{8}$	A1	1.1b
	Deduces $k < 0, k > \frac{15}{8}$	A1	2.2a
		(6)	