Question	Scheme	Marks	AOs
12(a)	$V=\pi r^2 h=355 \Rightarrow h=\dfrac{355}{\pi r^2}$ $\left(\text{or } rh=\dfrac{355}{\pi r} \text{ or } \pi rh=\dfrac{355}{r}\right)$	B1	1.1b
	$C=0.04\left(\pi r^2+2\pi rh\right)+0.09\left(\pi r^2\right)$	M1	3.4
	$C=0.13\pi r^2+0.08\pi rh=0.13\pi r^2+0.08\pi r\left(\dfrac{355}{\pi r^2}\right)$	dM1	2.1
	$C=0.13\pi r^2+\dfrac{28.4}{r}$ *	A1*	1.1b
		(4)	
(b)	$\dfrac{dC}{dr}=0.26\pi r-\dfrac{28.4}{r^2}$	M1 A1	3.4 1.1b
	$\dfrac{dC}{dr}=0 \Rightarrow r^3=\dfrac{28.4}{0.26\pi} \Rightarrow r=...$	M1	1.1b
	$r=\sqrt[3]{\dfrac{1420}{13\pi}}=3.26...$	A1	1.1b
		(4)	
(c)	$\left(\dfrac{d^2C}{dr^2}=\right)\ 0.26\pi+\dfrac{56.8}{r^3}=0.26\pi+\dfrac{56.8}{\text{"}3.26\text{"}^3}$	M1	1.1b
	$\left(\dfrac{d^2C}{dr^2}=\right)\ (2.45..)>0$ Hence minimum (cost)	A1	2.4
		(2)	
(d)	$C=0.13\pi\left(\text{"}3.26\text{"}\right)^2+\dfrac{28.4}{\text{"}3.26\text{"}}$	M1	3.4
	$(C=)13$	A1	1.1b
		(2)	
		(12 marks)	

Notes

(a)

B1: Correct expression for h or rh or πrh in terms of r. This may be implied by their later substitution.

M1: Scored for the sum of the three terms of the form $0.04...r^2$, $0.09...r^2$ and $0.04\times...rh$

The $0.04\times...rh$ may be implied by eg $0.04\times...r\times\dfrac{355}{\pi r^2}$ if h has already been replaced

dM1: Substitutes h or rh or πrh into their equation for C which must be of an allowable form (see above) to obtain an equation connecting C and r.
It is dependent on a correct expression for h or rh or πrh in terms of r

A1*: Achieves given answer with no errors. Allow Cost instead of C but they cannot just have an expression.
As a minimum you must see
- the separate equation for volume
- the two costs for the top and bottom separate before combining
- a substitution before seeing the $\frac{28.4}{r}$ term

Eg $355 = \pi r^2 h$ and $C = 0.04\pi r^2 + 0.09\pi r^2 + 0.04 \times 2\pi rh = 0.13\pi r^2 + 0.08\pi \times \left(\frac{355}{\pi r}\right)$

(b)

M1: Differentiates to obtain at least $r^{-1} \rightarrow r^{-2}$

A1: Correct derivative.

M1: Sets $\frac{dC}{dr} = 0$ and solves for r. There must have been some attempt at differentiation of the equation for C ($...r^2 \rightarrow ...r$ or $...r^{-1} \rightarrow ...r^{-2}$) Do not be concerned with the mechanics of their rearrangement and do not withhold this mark if their solution for r is negative

A1: Correct value for r. Allow exact value or awrt 3.26

(c)

M1: Finds $\frac{d^2C}{dr^2}$ at their (positive) r or considers the sign of $\frac{d^2C}{dr^2}$.
This mark can be scored as long as their second derivative is of the form $A + \frac{B}{r^3}$ where A and B are non zero

A1: Requires
- A correct $\frac{d^2C}{dr^2}$
- Either
  - deduces $\frac{d^2C}{dr^2} > 0$ for $r > 0$ (without evaluating). There must be some minimal explanation as to why it is positive.
  - substitute their positive r into $\frac{d^2C}{dr^2}$ without evaluating and deduces $\frac{d^2C}{dr^2} > 0$ for $r > 0$
  - evaluate $\frac{d^2C}{dr^2}$ (which must be awrt 2.5) and deduces $\frac{d^2C}{dr^2} > 0$ for $r > 0$

(d)

M1: Uses the model and their positive r found in (b) to find the minimum cost. Their r embedded in the expression is sufficient. May be seen in (b) but must be used in (d).

A1: $(C =)\ 13$ ignore units