## Question

Scheme

| 13(a) | ( $V=$ ) $\pi r^{2} h=400$ | B1 | 1.1b |
| :---: | :---: | :---: | :---: |
|  | $A=2 \pi r^{2}+2 \pi r h$ | B1 | 1.1b |
|  | $h=\frac{400}{\pi r^{2}} \Rightarrow A=2 \pi r^{2}+2 \pi r\left(\frac{400}{\pi r^{2}}\right)$ | M1 | 1.1b |
|  | $A=2 \pi r^{2}+\frac{800}{r} *$ | A1* | 1.1b |
|  |  | (4) |  |
| (b) | Attempts to differentiate $A=2 \pi r^{2}+\frac{800}{r}$ with respect to $r$ $\frac{\mathrm{d} A}{\mathrm{~d} r}=4 \pi r-800 r^{-2}$ | $\begin{gathered} \text { M1 } \\ \text { A1 } \end{gathered}$ | $\begin{aligned} & 3.1 \mathrm{~b} \\ & 1.1 \mathrm{~b} \end{aligned}$ |
|  | Sets $\frac{\mathrm{d} A}{\mathrm{~d} r}=0 \Rightarrow r^{3}=\frac{200}{\pi}$ | dM1 | 1.1b |
|  | $\Rightarrow r=\sqrt[3]{\frac{200}{\pi}}(\mathrm{~cm})$ | A1 | 1.1b |
|  |  | (4) |  |
| (c) | Finds $\frac{\mathrm{d}^{2} A}{\mathrm{~d} r^{2}}=4 \pi+1600 r^{-3}$ at $r=\sqrt[3]{\frac{200}{\pi}}$ | M1 | 1.1b |
|  | $\frac{\mathrm{d}^{2} A}{\mathrm{~d} r^{2}}=(+37.7)>0$ hence minimum (surface area). | A1ft | 2.4 |
|  |  | (2) |  |
| (d) | Substitutes $r=\sqrt[3]{\frac{200}{\pi}}$ in $A=2 \pi r^{2}+\frac{800}{r}$ | M1 | 1.1b |
|  | Minimum surface area $=$ awrt $301\left(\mathrm{~cm}^{2}\right)$ | A1ft | 1.1b |
|  |  | (2) |  |

(12 marks)

## Notes:

(a)

B1: Correct equation for volume: $\pi r^{2} h=400$
B1: Correct formula for surface area in terms of the radius and height: $A=2 \pi r^{2}+2 \pi r h$
M1: Rearranges $\pi r^{2} h=400$ to $h=\frac{400}{\pi r^{2}}$ and substitutes in to $h$ in their formula for the surface area A1*: cso.
(b)

M1: Attempts to differentiate $A=2 \pi r^{2}+\frac{800}{r}$ with respect to $r$. Look for $\left(\frac{\mathrm{d} A}{\mathrm{~d} r}=\right) \ldots r \pm \ldots r^{-2}$
A1: $\left(\frac{\mathrm{d} A}{\mathrm{~d} r}=\right) 4 \pi r-800 r^{-2}$ Condone $\frac{\mathrm{d} A}{\mathrm{~d} r}$ appearing as $\frac{\mathrm{d} y}{\mathrm{~d} x}$ or being absent.
$\mathbf{d M 1 : ~ S e t s ~ t h e i r ~} \frac{\mathrm{d} A}{\mathrm{~d} r}=0$ and arrives at $r^{3}=k, k>0 . \frac{\mathrm{d} A}{\mathrm{~d} r}$ must have been of the form $\ldots r \pm \ldots r^{-2}$ A1: $r=\sqrt[3]{\frac{200}{\pi}}$ or exact equivalent. Condone omission of units or use of incorrect units. Note $r=3.99$ to s.f.
(c)

M1: Finds $\frac{\mathrm{d}^{2} A}{\mathrm{~d} r^{2}}$ following on from their $\frac{\mathrm{d} A}{\mathrm{~d} r}$ (which must be of equivalent difficulty) and attempts to find its value or sign at their $r$
A1ft: $\frac{\mathrm{d}^{2} A}{\mathrm{~d} r^{2}}=(+37.7)>0$ hence minimum (surface area).
Alternatively, $\frac{\mathrm{d}^{2} A}{\mathrm{~d} r^{2}}=4 \pi+1600 r^{-3}>0$ as $+\mathrm{ve}++\mathrm{ve}>0$ as $r>0$.
Requires a correct calculation or expression, a correct statement, and a correct conclusion.
Follow through on their $r(r>0)$ and their $\frac{\mathrm{d}^{2} A}{\mathrm{~d} r^{2}}$.
$\frac{d^{2} A}{d r^{2}}$ must be used for this mark to meet the demand of the question.
(d)

M1: For a correct method for finding $A=$ from their solution to $\frac{\mathrm{d} A}{\mathrm{~d} r}=0$
May be implied by correct final answer. Do not accept attempts using negative values of $r$.
A1ft: Minimum surface area $=$ awrt $301\left(\mathrm{~cm}^{2}\right)$ Condone omission of units or use of incorrect units.

