12. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

The temperature, $\theta\,°C$, of a cup of coffee as it cools is modelled by the equation

$$\theta = 22 + 64e^{-\frac{3}{32}t} \qquad t \geqslant 0$$

where t is the time in minutes since the cup of coffee was prepared.

(a) State the starting temperature of the coffee according to the model.

(1)

(b) Find the rate at which the coffee is cooling 10 minutes after it was prepared.

(3)

(c) Determine, according to the model, the length of time that the temperature of the coffee is between $40\,°C$ and $60\,°C$. Give your answer to the nearest second.

(4)

Two hours after it was prepared, the temperature of the coffee is $20.8\,°C$.

(d) Evaluate the model in light of this information.

(1)

An insulated cup is designed to decrease the rate at which the coffee cools down.

(e) Explain how the model could be refined when this cup is used.

(1)