Question	Scheme	Marks	AOs
17 (a)	Provides a counter example with a reason. e.g., $6^3 - 1^3 = 215$ which is a multiple of 5	B1	2.4
		(1)	
(b)	States or uses, e.g., $2n$ and $2n+2$ **or** $2n+2$ and $2n+4$	M1	2.1
	Attempts $(2n+2)^3 - (2n)^3 = 8n^3 + 24n^2 + 24n + 8 - 8n^3$ leading to a quadratic.	dM1	1.1b
	$= 24n^2 + 24n + 8$	A1	1.1b
	$24n^2 + 24n + 8 = 8(3n^2 + 3n + 1)$ So $q^3 - p^3$ is a multiple of 8	A1	2.1
		(4)	
			(5 marks)

Notes:

(a)

B1: Provides a counter example with a reason. There is no need to state "not true".

e.g., $7^3 - 2^3 = 335$ which divides by 5 {exactly}.

It is sufficient to have, e.g., $9^3 - 4^3 = 665$ and $\frac{665}{5} = 133$

Here q must be greater than p and both must be natural numbers, not 0 or negatives.
Note that any pair of positive integers n and $n+5k$ will provide a counter example, but $q^3 - p^3$ must be evaluated correctly, and if they divide by 5 this also needs to be correct.

(b)

M1: For the key step in stating the algebraic form of consecutive even numbers.
See main scheme for examples. They might be used either way round for this mark.

dM1: Attempts $(2n+2)^3 - (2n)^3 = ...$ condoning slips but must lead to a quadratic.

Alternatively, $(2n+2)^3 - (2n)^3 = 2^3\{(n+1)^3 - n^3\}$

May be subtracted the wrong way round for this mark as below.
$(2n)^3 - (2n+2)^3 = ...$ but this will score M1dM1A0A0

A1: e.g., $(2n+2)^3 - (2n)^3 = 24n^2 + 24n + 8$ **or** $(2n+4)^3 - (2n+2)^3 = 24n^2 + 72n + 56$

or $(2n+2)^3 - (2n)^3 = 8\{(n+1)^3 - n^3\}$ **or** $(2n)^3 - (2n-2)^3 = 24n^2 - 24n + 8$ etc.

Must come from correct work and the algebra will need checking carefully.

A1: For a full and rigorous proof showing all necessary steps including:

- correct quadratic expression for $q^3 - p^3$ for their even numbers, e.g., $24n^2 + 24n + 8$
- reason e.g., $24n^2 + 24n + 8 = 8(3n^2 + 3n + 1)$ **or**, e.g., in $24n^2 + 24n + 8$ the coefficients are all multiples of 8
- minimal conclusion, "hence true"

Alt 1:

If the even numbers are set as n and $n + 2$ there must be sufficient work seen before marks can be awarded.

e.g.,

M1dM1: $n = 2k \Rightarrow (n+2)^3 - n^3 = ...n^2 + ...n + ... = ...(2k)^2 + ...(2k) + ...$

A1: $= 24k^2 + 24k + 8$

A1: $= 8\left(3k^2 + 3k + 1\right)$ so $q^3 - p^3$ is a multiple of 8

Alt 2:

If they just use any two even numbers, e.g., 2a and 2b, **or** 2m and 2n + 2 then they will score as follows:

M1: $(2a)^3 - (2b)^3$ Condone missing brackets if recovered.

dM1: $= ...a^3 - ...b^3$

A1: $= 8a^3 - 8b^3$ Note $8\left(a^3 - b^3\right)$ would imply this mark.

A1: $= 8\left(a^3 - b^3\right)$ so $q^3 - p^3$ is a multiple of 8 if q and p are {any two} even {numbers}

and hence $q^3 - p^3$ is a multiple of 8 if q and p are *consecutive* even numbers