

Question	Scheme	Marks	AOs
2(a)	$(f(4)=) 2 \times 4^3 - 3a \times 4^2 + 4b + 8a = 0$	M1	1.1b
	$128 + 4b = 40a \Rightarrow 32 + b = 10a *$	A1*	1.1b
		(2)	
(b)	$f(2) = 2 \times 2^3 - 3a \times 2^2 + 2b + 8a = 0 \Rightarrow 8 + b = 2a$	M1	1.1b
	Solve simultaneously $\Rightarrow a = \dots$ or $\Rightarrow b = \dots$	dM1	2.1
	$a = 3$ or $b = -2$ or $k = 3$	A1	1.1b
	$(f(x)=) (2x+3)(x-4)(x-2)$	A1	1.1b
		(4)	
(c)(i)	3	B1	1.1b
(ii)	12	B1ft	2.2a
		(2)	

(8 marks)

Notes

(a)

M1: Attempts $f(4) = 0$ leading to an equation in a and b only. Condone slips. The $= 0$ may be implied by further work for this mark. Attempts using algebraic division score M0A0.

A1*: Simplifies and rearranges to the given answer with no errors. There must be at least one intermediate stage of working between their first expression or equation and the given answer and $= 0$ must be correctly seen at some point in their solution or at the start e.g. stating $f(4) = 0$. Isw if they achieve the given answer but then attempt to make a or b the subject.

Minimum acceptable is e.g. $128 - 48a + 4b + 8a \Rightarrow 128 - 40a + 4b = 0 \Rightarrow 32 + b = 10a$

Note: $128 - 48a + 4b + 8a \Rightarrow 128 + 4b = 40a \Rightarrow 32 + b = 10a$ is M1A0* (we do not see $= 0$ correctly at some point or e.g. $f(4) = 0$)

(b) **Note that there are many different equations which can be formed.**

Sight of $a = 3$ or $b = -2$ or $k = 3$ scores the first 3 marks BUT answers with no working – send to review

M1: Attempts $f(2) = 0$ to form another equation in a and b . Does not need to be simplified. Condone slips substituting in 2 (but not if there is a clear intention to substitute in -2) and $= 0$ may be implied by further work. Accept attempts to divide algebraically by $x - 2$: e.g. condone slips but they must have a quadratic quotient $2x^2 \pm (\dots a \pm \dots)x$ and proceed as far as a remainder of the form $\dots a \pm \dots b \pm \dots$ which is then set equal to 0.

$$\begin{array}{r}
 2x^2 + (-3a+4)x \quad + (b-6a+8) \\
 \hline
 \text{e.g. } x-2 \left) \begin{array}{r} 2x^3 \quad -3ax^2 \quad +bx \quad +8a \\ 2x^3 \quad -4x^2 \\ \hline (-3a+4)x^2 \quad +bx \\ (-3a+4)x^2 + (6a-8)x \\ \hline (b-6a+8)x \quad +8a \\ (b-6a+8)x - 2b + 12a - 16 \\ \hline -4a + 2b + 16 = 0 \end{array}
 \end{array}$$

You may also see an attempt at the grid method:

	$2x^2$	$+(4-3a)x$	$+(8-6a+b)$
x	$2x^3$	$+(4-3a)x^2$	$+(8-6a+b)x$
-2	$-4x^2$	$(-8+6a)x$	$-16+12a-2b$
$-16+12a-2b = 8a$			

In this method condone slips but they need to proceed as far as a quadratic quotient of the form $2x^2 \pm (...a \pm ...)x$ and the bottom right cell of the form $...a \pm ...b \pm ...$ which is then set equal to $8a$.

Look out for other valid methods such as

- multiplying out $f(x) = (2x+k)(x-4)(x-2)$ to achieve a cubic where the coefficients of x^2 and x , and the constant term are all in terms of k , and equating coefficients to form two more simultaneous equations
- algebraically dividing $2x^3 - 3ax^2 + bx + 8a$ by $x^2 - 6x + 8$ and setting their remainder equal to zero. Look for a quotient of the form $2x \pm ...a \pm ...$
- substituting $b = 10a - 32$ into $f(x)$ so that the polynomial coefficients are in terms of a only (or equivalently all in b only) and using $f(2) = 0$ to form an equation in one variable. Send to review if you are unsure.

dM1: Attempts to solve their equations simultaneously to find a value for a or b (or they may proceed directly to finding a value for k). This may be done on a calculator. You do not need to check their method for solving. It is dependent on the previous method mark.

A1: $a = 3$ or $b = -2$ or $k = 3$

A1: $(f(x) =) (2x+3)(x-4)(x-2)$ (All on one line) (Stating the values of a , b and k alone does not score this mark). Allow to be scored if seen in (c).

Alternative Further Maths method using the sum and product of the roots

M1: $\alpha + \beta + \gamma = \frac{3a}{2} \Rightarrow 4 + 2 - \frac{k}{2} = \frac{3a}{2}$ or $\alpha\beta\gamma = -\frac{8a}{2} \Rightarrow 4 \times 2 \times \left(-\frac{k}{2}\right) = -\frac{8a}{2}$

A1: $\alpha + \beta + \gamma = \frac{3a}{2} \Rightarrow 4 + 2 - \frac{k}{2} = \frac{3a}{2}$ and $\alpha\beta\gamma = -\frac{8a}{2} \Rightarrow 4 \times 2 \times \left(-\frac{k}{2}\right) = -\frac{8a}{2}$

(Less likely but possible to see $\alpha\beta + \beta\gamma + \alpha\gamma = \frac{b}{2} \Rightarrow 4 \times 2 + 2 \times \left(-\frac{k}{2}\right) + 4 \times \left(-\frac{k}{2}\right) = \frac{b}{2}$)

dM1A1A1: See notes above

- (c)
 (i)
 B1: 3 (listing the actual roots only is B0)

(ii)
 B1ft: 12 only Follow through on their $2x+k \Rightarrow -\frac{3k}{2}$ if $k < -8$