Question	Scheme	Marks	AOs
14	Sets up the proof by exploring when $n=2k$ or $n=2k+1$ e.g. $(2k)^2+5(2k)=...k^2+...k$ **or** $(2k+1)^2+5(2k+1)=...k^2+...k+...$	M1	1.1b
	e.g. $4k^2+10k$ **or** $4k^2+14k+6$ and shows or gives a reason why the expression is even (see notes)	A1	2.2a
	Explores when $n=2k$ **and** $n=2k+1$ eg $(2k)^2+5(2k)=...k^2+...k$ **and** $(2k+1)^2+5(2k+1)=...k^2+...k+...$	dM1	2.1
	e.g. $4k^2+10k$ **and** $4k^2+14k+6$ and shows or gives a reason why both of the expressions are even (see notes) hence n^2+5n is even for all $n(\in \mathbb{N})$ (or equivalent)	A1*	2.4
			(4 marks)

Notes

Main scheme algebraic method using e.g. $n=2k$ and $n=2k\pm1$
You will need to look at both cases and mark the one which is fully correct first.
Allow a different variable to *k* and may be different letters for odd and even.
Condone use of *n* as a variable for the first three marks.
There should be no errors in the algebra for the A marks but allow e.g. invisible brackets to be "recovered".

M1: Sets up the proof by exploring when n is odd or even e.g. $n=2k$ or $n=2k+1$ (or equivalent), and either expands and achieves a quadratic expression (which may be unsimplified) or allow to factorise e.g. $2k(2k+5)$ or e.g. $(2k+2)(2k+7)$

Condone slips. e.g. $2k(2k+5)=2k^2+10k$ or slips when collecting terms.

A1: Correct quadratic expression (which may be unsimplified) for n^2+5n for either odds or evens and shows or gives a reason why the expression is even. They must have fully multiplied out or the quadratic expression must be factorised completely.

e.g. $4k^2+10k=2(2k^2+5k)$ (which is even)

e.g. $4k^2+14k+6=2(2k^2+7k+3)$ (which is even)

e.g. $\dfrac{4k^2+10k}{2}=2k^2+5k$ (hence even)

e.g. "2 is a factor of both terms", "all divisible by 2" (so even)
If a reason is given as well as an algebraic expression it must be correct
e.g. $4k^2+10k=2(2k^2+5k)$ so even as can be multiplied by 2 can score M1A1
but $\dfrac{4k^2+10k}{2}=2k^2+5k$ so it can be divided by 2 so even is M1A0 (needs to say divisible by 2)
Do not isw if they simplify their quadratic incorrectly.
Note that they do not have to state that the expression is even if they conclude for all cases at the end.

dM1: Explores when n is odd and when n is even leading to two quadratic expressions (may be factorised) for when $n=2k$ and $n=2k+1$ (or equivalent) (see first M1 for guidance)

A1*: Requires

- correct quadratic expression for n^2+5n for both odds and evens
- shows or gives a reason for each why the expressions are even (see first A1 for guidance)
- makes a concluding overall statement. "Hence n^2+5n is even for all $n(\in \mathbb{N})$" (or equivalent).

Note that if they have stated for each separate case that the expression is even then allow minimal statements of "hence proven", "statement proved", "QED", tick
Do not isw this mark if they simplify their quadratic incorrectly.

	n^2+5n
$2k-3$	$4k^2-2k-6$
$2k-2$	$4k^2+2k-6$
$2k-1$	$4k^2+6k-4$
$2k$	$4k^2+10k$
$2k+1$	$4k^2+14k+6$
$2k+2$	$4k^2+18k+14$
$2k+3$	$4k^2+22k+24$

Alternative methods:
Algebraic with logic example

e.g. $n^2+5n=n(n+5)$

When n is odd then $n+5$ is even so odd x even is even
When n is even then $n+5$ is odd so even x odd is even

Both cases must be considered to score any marks and scores SC 1010 if fully correct

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Further Maths method (proof by induction) – you may see these but please send to review for TLs or above to mark

M1: Assumes true for $n=k$, substitutes $n=k+1$ into n^2+5n, multiplies out the brackets and attempts to simplify to a quadratic expression (which may be unsimplified)
e.g. k^2+7k+6 Condone arithmetical slips

A1: $\left(\mathrm{f}(k+1)=3k^2+3k+1+6(k+1)=\right)\ k^2+5k+2k+6=\mathrm{f}(k)+2(k+3)$
which is even + even = even

dM1: Attempts to substitute $n=1\Rightarrow\ 1^2+5\times1=6$ (which is true) (Condone arithmetical slips evaluating)

A1*: Explains that

- it is true when $n=1$
- if it is true for $n=k$ then it is true for $n=k+1$
- therefore it is true for all $n(\in \mathbb{N})$

Solutions via just logic (no algebraic manipulation) scores 0 marks.

e.g.

If n is odd, then n^2+5n is $odd^2+odd\times odd=odd+odd=even$

If n is even, then n^2+5n is $even^2+odd\times even=even+even=even$