| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 3(a) | $\left(\lvert\overrightarrow{OB}\rvert=\right)\sqrt{(...)^2+(\pm2)^2}$ or $\left(\lvert\overrightarrow{OB}\rvert=\right)\sqrt{(\pm6)^2+(...)^2}$ or $\left(\lvert\overrightarrow{OB}\rvert^2=\right)(...)^2+(\pm2)^2$ or $\left(\lvert\overrightarrow{OB}\rvert^2=\right)(\pm6)^2+(...)^2$ | M1 | 1.1b |
| | $\left(\lvert\overrightarrow{OB}\rvert=\right)\sqrt{(\pm2)^2+(\pm6)^2}$ | dM1 | 1.1b |
| | $\sqrt{40}$ or $2\sqrt{10}$ | A1 | 1.1b |
| | | **(3)** | |
| (b) | $\lvert\overrightarrow{OA}\rvert=\sqrt{73}$ or $\lvert\overrightarrow{AB}\rvert=\sqrt{29}$ | B1 | 1.1b |
| | $40=73+29-2\sqrt{73}\sqrt{29}\cos OAB \Rightarrow \cos OAB=\dfrac{73+29-40}{2\sqrt{73}\sqrt{29}}$ $\Rightarrow OAB=\cos^{-1}\left(\dfrac{73+29-40}{2\sqrt{73}\sqrt{29}}\right)$ or $\cos OAB=\dfrac{73+29-40}{2\sqrt{73}\sqrt{29}}\Rightarrow OAB=...$ | M1 | 3.1a |
| | $OAB=47.6°$ | A1 | 1.1b |
| | | **(3)** | |
| | | **(6 marks)** | |

**Notes:**

**Note that marks in (a) can be scored in (b) as long as they are not contradictory.**

**Note that they are not asked for $\overrightarrow{OB}$ in (a) but $\lvert\overrightarrow{OB}\rvert$. As such, all they need are the magnitudes of the components of $\overrightarrow{OB}$ to find $\lvert\overrightarrow{OB}\rvert$ so you can ignore if $\overrightarrow{OB}$ is correct or not in both parts and full marks can be awarded even if there are sign errors in their $\overrightarrow{OB}$ if they write it as a vector.**

**(a)**

**M1:** Attempts $\lvert\overrightarrow{OB}\rvert$ or $\lvert\overrightarrow{OB}\rvert^2$ with one component correct and the other component non-zero.

Allow $\sqrt{(\pm2)^2+(...)^2}$ or $\sqrt{(\pm6)^2+(...)^2}$ or $(...)^2+(\pm2)^2$ or $(\pm6)^2+(...)^2$

and condone e.g. $-2^2$ or $-6^2$

**But** it must clearly not be an attempt at e.g. $\lvert\overrightarrow{AB}\rvert$ e.g. $\sqrt{5^2+2^2}$

**dM1:** Complete and correct method for $\lvert\overrightarrow{OB}\rvert$ i.e. $\lvert\overrightarrow{OB}\rvert=\sqrt{(\pm2)^2+(\pm6)^2}$

**A1:** $\sqrt{40}$ or $2\sqrt{10}$ only but isw if they then use decimals.

**Beware in (b) that assuming *OAB* is right angled can give answers that look approximately correct e.g.** $\sin OAB = \frac{OB}{AB} = \frac{\sqrt{40}}{\sqrt{73}} \Rightarrow OAB = \sin^{-1}\frac{\sqrt{40}}{\sqrt{73}} = 47.75...$ **but is an incorrect method.**

**In (b) mark the method that is most successful.**

**(b)** **Way 1: Cosine rule**

**B1:** Finds either of $\left|\overrightarrow{OA}\right| = \sqrt{73}$ or $\left|\overrightarrow{AB}\right| = \sqrt{29}$ allow for sight of these values even if not associated with a vector. They may be seen on a diagram or embedded in an attempt at the cosine rule. May be implied by decimal values (see diagram)

**M1:** A complete and correct method for finding angle *OAB* with their *OA*, *OB* and *AB*.
Correct attempt at the cosine rule leading to a value for angle *OAB* using arccos.
Following the correct use of the cosine rule, if a value for angle *OAB* is just written down or there is no evidence of arccos, you may need to check.
Following the correct use of the cosine rule, sufficient evidence could be e.g.
$\cos OAB = k \Rightarrow OAB = \cos^{-1} k = ...$

**A1:** awrt $47.6°$ Condone omission of degrees symbol.

**(b)** **Way 2: Right angled triangles**

**B1:** Finds any of $\tan^{-1}\left(\frac{8}{3}\right) = 69.4°$ , $\tan^{-1}\left(\frac{2}{5}\right) = 21.8°$ , $\tan^{-1}\left(\frac{3}{8}\right) = 20.6°$ , $\tan^{-1}\left(\frac{5}{2}\right) = 68.2°$
May be implied.

**M1:** A complete and correct method for finding angle *OAB*.
e.g. attempts $\tan^{-1}\left(\frac{5}{2}\right) - \tan^{-1}\left(\frac{3}{8}\right)$ or $\tan^{-1}\left(\frac{8}{3}\right) - \tan^{-1}\left(\frac{2}{5}\right)$ or $90° - \tan^{-1}\left(\frac{3}{8}\right) - \tan^{-1}\left(\frac{2}{5}\right)$
leading to a value for angle *OAB*.

**A1:** awrt $47.6°$ Condone omission of degrees symbol.

**(b)** **Way 3: Scalar product**

**B1:** Finds $\overrightarrow{AO} \bullet \overrightarrow{AB} = \begin{pmatrix} -3 \\ 8 \end{pmatrix} \bullet \begin{pmatrix} -5 \\ 2 \end{pmatrix} = 15 + 16 = 31$
Allow $\pm\overrightarrow{AO} \bullet \pm\overrightarrow{AB} = \pm 31$

**M1:** A complete and correct method for finding angle *OAB* with their *OA*, *OB* and *AB*.
e.g. $31 = \left|\overrightarrow{AO}\right|\left|\overrightarrow{AB}\right| \cos OAB = \sqrt{3^2 + 8^2}\sqrt{5^2 + 2^2} \cos OAB \Rightarrow \cos OAB = \frac{31}{\sqrt{73}\sqrt{29}} \Rightarrow OAB = ...$
If e.g. $\overrightarrow{OA} \bullet \overrightarrow{AB}$ is attempted then they need to find e.g. $OAB = 180° - \cos^{-1}\frac{-31}{\sqrt{73}\sqrt{29}}$
Following the correct use of the scalar product, if a value for angle *OAB* is just written down or there is no evidence of arccos, you may need to check.

**A1:** awrt $47.6°$ Condone omission of degrees symbol.

**There may be other methods for finding angle *OAB*.**

**For reference:**