| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 9 | $x^2-3px+5q+4=0$ | | |
| | $(3p)^2-4(5q+4)$ | B1 | 1.1b |
| | $9p^2-20q-16=0$<br>$(3p+4)(3p-4)=20q$ | M1 | 3.1a |
| | $q=\frac{1}{20}(3p+4)(3p-4)$ * | A1* | 2.1 |
| | | **(3)** | |

**Note we are marking this B1M1A1 not M1M1A1**

**B1:** **Correct** expression for the discriminant e.g. $(-3p)^2-4(5q+4)$ or $(3p)^2-4(5q+4)$

Do **not** condone missing brackets around the "3$p$" or $5q + 4$ unless they are recovered **but** missing brackets around the "3$p$" must be recovered before using the difference of 2 squares below. E.g. $3p^2-16=(3p+4)(3p+4)$ scores M0 below.

Allow to be seen embedded in at attempt at the quadratic formula.

Allow for $9p^2-20q-16$ just written down.

**M1:** Sets their discriminant in terms of $p$ and $q$ $=0$ with the correct use of $\alpha p^2-\beta=\left(\sqrt{\alpha}p+\sqrt{\beta}\right)\left(\sqrt{\alpha}p-\sqrt{\beta}\right)$ so $9p^2-16=20q=(9p+12)(9p-12)$ scores M0 even if "recovered"

**A1*:** cso Arrives at $q=\frac{1}{20}(3p+4)(3p-4)$ with no errors seen.

Or expands $q=\frac{1}{20}(3p+4)(3p-4)$ and compares with their q and gives a (minimal) conclusion. Any missing brackets must be recovered before reaching the printed answer.

A correct intermediate line must be seen, such as $q=\frac{9p^2-16}{20}$ or $20q=(3p+4)(3p-4)$

| | | | |
|---|---|---|---|
| | **Alternative 1: Completes the square** | | |
| | $x^2-3px+5q+4=\left(x-\frac{3p}{2}\right)^2-\frac{9p^2}{4}+5q+4$ | B1 | 1.1b |
| | Equal roots $\Rightarrow -\frac{9p^2}{4}+5q+4=0$ <br> $\Rightarrow 20q=9p^2-16=(3p+4)(3p-4)$ | M1 | 3.1a |
| | $q=\frac{1}{20}(3p+4)(3p-4)$* | A1* | 2.1 |

**Note we are marking this B1M1A1 not M1M1A1**

**B1:** **Correct** completion of the square $x^2-3px+5q+4=\left(x-\frac{3p}{2}\right)^2-\left(\frac{3p}{2}\right)^2+5q+4$

Do **not** condone missing brackets around the $\frac{3p}{2}$ unless they are recovered.

**M1:** Sets their $-\left(\frac{3p}{2}\right)^2+5q+4$ which must be in terms of $p$ and $q$ equal to 0 with the correct use of $\alpha p^2-\beta=\left(\sqrt{\alpha}p+\sqrt{\beta}\right)\left(\sqrt{\alpha}p-\sqrt{\beta}\right)$ so $9p^2-16=20q=(9p+12)(9p-12)$ scores M0 even if "recovered"

**A1*:** cso Arrives at $q=\frac{1}{20}(3p+4)(3p-4)$ with no errors seen.

Any missing brackets must be recovered before reaching the printed answer.

A correct intermediate line must be seen, such as $q=\frac{9p^2-16}{20}$ or $20q=(3p+4)(3p-4)$

| | | | |
|---|---|---|---|
| | **Alternative 2: Calculus** | | |
| | $y=x^2-3px+5q+4\Rightarrow\frac{dy}{dx}=2x-3p$ | B1 | 1.1b |
| | $2x-3p=0\Rightarrow x=\frac{3p}{2}\Rightarrow\frac{9p^2}{4}-\frac{9p^2}{2}+5q+4=0$ <br> $\Rightarrow 20q=9p^2-16=(3p+4)(3p-4)$ | M1 | 3.1a |
| | $q=\frac{1}{20}(3p+4)(3p-4)$* | A1* | 2.1 |

**Note we are marking this B1M1A1 not M1M1A1**

**B1:** **Correct** derivative.

**M1:** Sets their $\frac{dy}{dx}$ which must be in terms of $p$ and $x$ equal to 0 to find $x$ in terms of $p$, substitutes into $x^2-3px+5q+4$ and with the correct use of $\alpha p^2-\beta=\left(\sqrt{\alpha}p+\sqrt{\beta}\right)\left(\sqrt{\alpha}p-\sqrt{\beta}\right)$ so $9p^2-16=20q=(9p+12)(9p-12)$ scores M0 even if "recovered"

**A1*:** cso Arrives at $q=\frac{1}{20}(3p+4)(3p-4)$ with no errors seen.

Any missing brackets must be recovered before reaching the printed answer.

A correct intermediate line must be seen, such as $q=\frac{9p^2-16}{20}$ or $20q=(3p+4)(3p-4)$

| | Alternative 3: Working backwards | | |
|---|---|---|---|
| | $q=\frac{1}{20}(3p+4)(3p-4)=\frac{1}{20}(9p^2-16)$ <br> $\Rightarrow x^2-3px+5q+4=x^2-3px+\frac{1}{4}(9p^2-16)+4=x^2-3px+\frac{9p^2}{4}$ | B1 | 1.1b |
| | $x^2-3px+\frac{9p^2}{4}=\left(x-\frac{3p}{2}\right)^2+...$ <br> or <br> $b^2-4ac=(3p)^2-4\times1\times\left(\frac{9p^2}{4}\right)=...$ | M1 | 3.1a |
| | e.g. $x=\frac{3p}{2}$ only so equal roots or $\left(x-\frac{3p}{2}\right)^2=0$ so equal roots <br> or <br> $b^2-4ac=0$ so equal roots | A1* | 2.1 |

**Notes:**

**Note we are marking this B1M1A1 not M1M1A1**

**B1:** Substitutes the given result into $x^2-3px+5q+4$ and simplifies to obtain $x^2-3px+\frac{9p^2}{4}$ oe e.g. $4x^2-12px+9p^2$

**M1:** Attempts to complete the square to obtain $x^2-3px+\frac{9p^2}{4}=\left(x-\frac{3p}{2}\right)^2+...$ oe

alternatively attempts the discriminant e.g. $(3p)^2-4\times1\times\left(\frac{9p^2}{4}\right)=...$

**A1*:** cso Fully correct work with sufficient working shown and no errors seen with a suitable reasoning and a conclusion e.g. "only 1 root so equal roots" or equivalent or obtains a zero discriminant and states e.g. "so equal roots" or equivalent.