In Q11 there are a lot of parts so mark positively and condone mislabelling if they are clearly doing the right thing for a particular part.

Question	Scheme	Marks	AOs
11(a)	$(V=)\frac{1}{2}\pi r^2 l = 90000\pi$	B1	1.1b
	$(A=)\pi r^2 + \pi rl$	B1	1.1b
	$l=\frac{180000}{r^2} \Rightarrow A=\pi r^2+\pi r\left(\frac{180000}{r^2}\right)$ or e.g. $\pi rl=\frac{180000\pi}{r} \Rightarrow A=\pi r^2+\frac{180000\pi}{r}$ or e.g. $rl=\frac{180000}{r} \Rightarrow A=\pi r^2+\pi\left(\frac{180000}{r}\right)$	M1	1.1b
	$(A=)\pi r^2+\frac{180000\pi}{r}$*	A1*	1.1b
		(4)	

(a) Condone use of e.g. h for l or e.g. R for r as long as the intention is clear.

B1: Correct equation seen for volume:

$$\frac{1}{2}\pi r^2 l = 90000\pi \text{ or } \pi r^2 l = 180000\pi \text{ or } \pi r^2 l = 2(90000\pi)$$

or

$$\frac{1}{2} r^2 l = 90000 \text{ or } r^2 l = 180000 \text{ or } r^2 l = 2(90000)$$

May not be implied by subsequent work.

B1: **Correct** expression for the area stated in terms of the radius (or diameter) and length:

e.g. $(A=)\pi r^2 + \pi rl$ but allow equivalent correct expression so that the "ends" are any one of:

πr^2	$\frac{1}{2}\pi r^2+\frac{1}{2}\pi r^2$	$\pi\left(\frac{d}{2}\right)^2$	$\frac{2\pi r^2}{2}$	$2\times 0.5\pi r^2$

And the curved surface area is any one of:

πrl	$\frac{1}{2}(2\pi rl)$	$\frac{\pi ld}{2}$

These may be seen separately before being combined at the end.

Condone a missing "$A=$" for this mark. Cannot be implied.

M1: Rearranges their $\frac{1}{2}\pi r^2 l = 90000\pi$ to e.g. $l=...$ or $\pi rl=...$ or $rl=...$ and substitutes in for e.g. l or πrl or rl in their formula for the area which must have at least 2 terms, all of which must be dimensionally correct, to obtain an expression in terms of r only. This mark cannot be implied and substitution needs to be shown, not just the printed answer written down. This may be seen combined at the end.

A1*: cso. The "$A=$" does not have to be seen. Ignore any units given.

All previous marks must be scored. Isw once a correct answer is reached from correct work.

(b)	$A=\pi r^2+\dfrac{180000\pi}{r}\Rightarrow\left(\dfrac{\mathrm{d}A}{\mathrm{d}r}=\right)2\pi r-180000\pi r^{-2}$	M1 A1	1.1b 1.1b
	$\dfrac{\mathrm{d}A}{\mathrm{d}r}=0\Rightarrow r^3=\dfrac{180000\pi}{2\pi}$	dM1	1.1b
	$\Rightarrow r=\sqrt[3]{90000}$ or $r=$ awrt 44.8 (cm)	A1	1.1b
		(4)	
(c)	Finds $\dfrac{\mathrm{d}^2A}{\mathrm{d}r^2}=2\pi+360000\pi r^{-3}$ at $r=44.8$	M1	3.1b
	$\dfrac{\mathrm{d}^2A}{\mathrm{d}r^2}=(+18.8)>0$ hence minimum (value of A)*	A1	2.4
		(2)	

(b)

M1: Attempts to differentiate $A=\pi r^2+\dfrac{180000\pi}{r}$ with respect to r.

Award for $\left(\dfrac{\mathrm{d}A}{\mathrm{d}r}=\right)\ldots r\pm\ldots r^{-2}$

A1: $\left\{\dfrac{\mathrm{d}A}{\mathrm{d}r}=\right\}2\pi r-180000\pi r^{-2}$ Condone $\dfrac{\mathrm{d}A}{\mathrm{d}r}$ appearing as $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ or being absent.

dM1: Sets their $\dfrac{\mathrm{d}A}{\mathrm{d}r}\ldots 0$ where "…" could be an inequality and arrives at a value for r via $r^{\pm 3}\ldots k, k>0$ where "…" could be an inequality.

May be implied by e.g. $r=\sqrt[3]{\ldots}$ or $r=\sqrt[-3]{\ldots}$ where "…" >0

They cannot just go from e.g. $2\pi r-180000\pi r^{-2}=0$ straight to a value for r.

A1: Awrt 44.8 or $\sqrt[3]{90000}$ Condone omission of units or use of incorrect units.

Do not allow e.g. ± 44.8 or e.g. $r>44.8$

(c)

M1: Attempts to find $\dfrac{\mathrm{d}^2A}{\mathrm{d}r^2}$ following on from their $\dfrac{\mathrm{d}A}{\mathrm{d}r}$ which includes $\ldots r\rightarrow$ constant and $\ldots r^{-2}\rightarrow\ldots r^{-3}$ and attempts to find its value with their r (may need to check) or considers its sign with reference to their r. Condone slips in substituting as long as the intention is clear.

Alt may consider the value of $\dfrac{\mathrm{d}A}{\mathrm{d}r}$ either side e.g. $\left(\dfrac{\mathrm{d}A}{\mathrm{d}r}\right)_{x=44}=-15.6\ldots,\left(\dfrac{\mathrm{d}A}{\mathrm{d}r}\right)_{x=45}=3.49\ldots$

or may consider the value of A either side e.g. $(A)_{x=44.8}=18927,\ (A)_{x=44}=18934,\ (A)_{x=45}=18928$

A1: Fully correct work and conclusion. This requires

- correct work in (b) with the correct value of r
- a correct $\dfrac{\mathrm{d}^2A}{\mathrm{d}r^2}$
- obtaining $\dfrac{\mathrm{d}^2A}{\mathrm{d}r^2}=$ awrt 19 (or 6π) (or truncated as 18) which is > 0 or e.g. $\dfrac{\mathrm{d}^2A}{\mathrm{d}r^2}>0$ as $r>0$
- a conclusion that it is the minimum

For the alt it would be for

- correct calculations
- e.g. gradient goes from negative to positive so minimum
- e.g. A is larger either side of 44.8 so minimum

(d)	Substitutes $r = 44.8$ into $A = \pi r^2 + \frac{180000\pi}{r}$ $\{= \text{awrt } 18900\}$ Or may be seen embedded in a (possibly incorrect) conversion e.g. substitutes $r = 44.8$ into $\frac{30}{100^2}\left(\pi r^2 + \frac{180000\pi}{r}\right)$ $\{= \text{awrt } 56.8\}$	M1	3.4
	Minimum cost = £56.78	A1	1.1b
		(2)	
(e)	Accept any sensible assumption, e.g., • The exact amount of metal required can be bought. • No metal goes to waste. • The trough can be made as a perfect (semicircular) cylinder. • It's possible to cut accurately Condone • Need to buy a fixed amount • You can't buy the exact amount of metal • There will be wastage due to the shape • No extra amount of metal is needed • There will be no errors when cutting the metal • The trough will be fully smooth/have no imperfections Do not allow e.g. • The thickness of the sheet is negligible	B1	3.5a
		(1)	
			(13 marks)

(d)

M1: For a correct method of finding A or $\frac{30A}{100^2}$ or e.g. $30A$ from their solution to $\frac{\mathrm{d}A}{\mathrm{d}r} = 0$

May be implied by e.g. 18928

Do not accept attempts using negative values of r.

A1: Minimum cost = £56.78 or £56.79 including units.

(e)

B1: See main scheme.

Do not accept reference to additional work required, such as labour costs, or materials required to weld the metal together.