Question	Scheme	Marks	AOs
13(a)	e.g. $6 = 31 - A\mathrm{e}^{-10k}, 11 = 31 - A\mathrm{e}^{-20k} \Rightarrow A\mathrm{e}^{-10k} = 25,\ A\mathrm{e}^{-20k} = 20$ $\Rightarrow \mathrm{e}^{10k} = \dfrac{25}{20}$	M1	3.1b
	e.g. $\mathrm{e}^{10k} = \dfrac{25}{20} \Rightarrow 10k = \ln\left(\dfrac{25}{20}\right) \Rightarrow k = \dfrac{1}{10}\ln\left(\dfrac{5}{4}\right)$	dM1	3.1a
	$k = 0.0223$ or $A = 31.3$	A1	1.1b
	$h = 31 - 31.3\mathrm{e}^{-0.0223t}$	A1	3.3
		(4)	

(a) Notes:

M1: Uses $h = 31 - A\mathrm{e}^{-kt}$ with $h = 6,\ t = 10$ and $h = 11,\ t = 20$ to create simultaneous equations and proceeds via correct work to eliminate A or k.

Condone the equations appearing as e.g. $6\mathrm{m} = 31 - A\mathrm{e}^{-10k}, 11\mathrm{m} = 31 - A\mathrm{e}^{-20k}$

Look for the overall method being correct but condone slips.

Examples:

Eliminates *A*:

- $A\mathrm{e}^{-10k} = 25,\ A\mathrm{e}^{-20k} = 20 \Rightarrow \mathrm{e}^{10k} = \dfrac{25}{20}$
- $A\mathrm{e}^{-10k} = 25,\ A\mathrm{e}^{-20k} = 20 \Rightarrow \ln A - 10k = \ln 25,\ \ln A - 20k = \ln 20 \Rightarrow 10k = \ln 25 - \ln 20$

Eliminates *k*:

- $A\mathrm{e}^{-10k} = 25,\ A\mathrm{e}^{-20k} = 20 \Rightarrow A\left(\mathrm{e}^{-10k}\right)^2 = 20 \Rightarrow A\left(\dfrac{25}{A}\right)^2 = 20$
- $A\mathrm{e}^{-10k} = 25,\ A\mathrm{e}^{-20k} = 20 \Rightarrow \ln A - 10k = \ln 25,\ \ln A - 20k = \ln 20 \Rightarrow \ln A = 2\ln 25 - \ln 20$

dM1: Proceeds to find a value for A or k using the correct order of operations but condoning slips.

A1: Achieves either k = awrt 0.0223 or A = awrt 31.3 (or A = awrt 31.2)

Note the exact values are $k = \dfrac{1}{10}\ln\dfrac{5}{4}$ or $A = \dfrac{125}{4}(31.25)$ which are acceptable.

A1: cso Proceeds correctly to $h = 31 - 31.2\mathrm{e}^{-0.0223t}$ or $h = 31 - 31.3\mathrm{e}^{-0.0223t}$

Exact answer is $h = 31 - \dfrac{125}{4}\mathrm{e}^{-\frac{1}{10}\ln\frac{5}{4}t}$ o.e., e.g., $h = 31 - 31.25\mathrm{e}^{\frac{1}{10}\ln\frac{4}{5}t}$ which is acceptable.

Finding *A* and *k* is insufficient for this mark as the question asks for a complete equation, however, allow this mark if the complete correct equation is seen subsequently.

(b)	31 m	B1	2.2a
		(1)	
(c)(i)	−0.3 m	B1ft	3.4
(ii)	The model is unsuitable for the early growth of the tree as it suggests that the tree had a negative height.	B1ft	3.5a
		(2)	

(b)

B1: cao 31 m and units are required. Must be seen as their answer to (b).
There must be no other values.

(c) (i)

B1ft: Accept $31-"A"$ with correct units. This must be evaluated correctly for their A.
Only penalise the omission of units **once** from an otherwise acceptable answer in (b) and (c)(i) and penalise it on the first occurrence.

(c) (ii)

B1ft: Concludes that the model is unsuitable because (the initial/early) **height** is **negative**
Follow through on their (c)(i) here but it must be negative.
Alternatively, allow that the model is suitable if they conclude that the depth of the seed is $"-0.3"$ m or e.g. suitable as it would be underground.

(d)(i)	$\dfrac{\mathrm{d}h}{\mathrm{d}t} = \ldots \mathrm{e}^{-kt}$	M1	1.1b
	$\left\{\dfrac{\mathrm{d}h}{\mathrm{d}t} =\right\}$ awrt $0.698\mathrm{e}^{-0.0223t}$	A1	1.1b
		(2)	
(d)(ii)	$"0.698\mathrm{e}^{-0.0223t}" = 0.3$	B1ft	3.4
	$\mathrm{e}^{-0.0223t} = \dfrac{0.3}{0.698} \Rightarrow -0.0223t = \ln\left(\dfrac{0.3}{0.698}\right)$	M1	3.1a
	$t = 37.9$	A1	1.1b
		(3)	

(d) (i)

M1: Achieves the form $\left\{\dfrac{\mathrm{d}h}{\mathrm{d}t} =\right\} a\mathrm{e}^{-kt}$. It is acceptable to use k and A for this mark and allow if $k < 0$ but must be consistent with their h.

A1: $\left\{\dfrac{\mathrm{d}h}{\mathrm{d}t} =\right\}$ awrt $0.698\mathrm{e}^{-0.0223t}$ Allow awrt $0.697\mathrm{e}^{-0.0223t}$ or awrt $0.696\mathrm{e}^{-0.0223t}$ and allow exact e.g. $\dfrac{25}{8}\ln\dfrac{5}{4}\mathrm{e}^{-\frac{1}{10}\ln\frac{5}{4}t}$

(d)(ii)

B1ft: Sets their $\dfrac{\mathrm{d}h}{\mathrm{d}t} = 0.3$ where $\dfrac{\mathrm{d}h}{\mathrm{d}t}$ is not their h.

M1: Uses their $\dfrac{\mathrm{d}h}{\mathrm{d}t} = \ldots$, and proceeds from $a\mathrm{e}^{-kt} = b$ where $ab > 0$ to $ct = \ln d$ where $d > 0$ using the correct order of operations but condoning slips.
Allow equivalent work e.g.
$0.696\mathrm{e}^{-0.0223t} = 0.3 \Rightarrow \ln 0.696 - 0.0223t = \ln 0.3 \Rightarrow -0.0223t = \ln 0.3 - \ln 0.696$

A1: cso awrt 37.7 or 37.8 or 37.9 but note that this may be scored if k is rounded in (a).
Ignore any units if given and just look for the value.