

Question	Scheme	Marks	AOs
15(a)	$n^3 + 4n$		
	Attempts $n^3 + 4n$ for any 2 natural numbers.	M1	1.1b
	$1^3 + 4 \times 1 = 5$ prime and e.g. $2^3 + 4 \times 2 = 16$ not prime \therefore sometimes true.	A1	2.4
		(2)	

Condone the use of n for e.g. k for both marks.

All methods require attempting $n^3 + 4n$ when $n = 1$ for full marks.

Way 1:

M1: Attempts $n^3 + 4n$ for any 2 natural numbers.

A1: Requires:

- obtains $n^3 + 4n = 5$ when $n = 1$ and states “prime” or “true” or ✓
- correct evaluation for any other natural number and states “not prime” or “composite” or “not true” or ✕
- states “sometimes true”

You can ignore any other incorrectly evaluated examples or e.g. use of negative numbers as long as these conditions are met.

Way 2 (factorisation):

M1: Attempts to factorise $n^3 + 4n = n(n^2 + 4)$

Allow for $n^3 + 4n = n(n^2 + \dots)$ or $n^3 + 4n = n(\dots + 4)$

A1: Requires:

- uses $n = 1$ and obtains $n^3 + 4n = 5$ and states “prime” or “true”
- correct factorisation and states $n^3 + 4n = n(n^2 + 4)$ is “composite” or “not prime” or “not true” (when $n > 1$)
- states “sometimes true”

Way 3 (odd/even):

M1: Attempts to substitute $n = 2k$ oe **or** $n = 2k + 1$ oe

A1: Requires:

- uses $n = 1$ and obtains $n^3 + 4n = 5$. Here the value for $n = 1$ might be found using $k = 1$ for $n = 2k - 1$ or $k = 0$ for $n = 2k + 1$ and states “prime” or “true”
- correctly factorises any one correct form for odd or even e.g. $8k^3 + 8k = 8k(k^2 + 1)$ and concludes “not prime” or “composite” or “not true” (only one form needed here)
- states “sometimes true”

(b)	$n^3 + 5n$		
	$n^3 + 5n = n(n^2 + 5)$	M1	3.1a
	Since $1^3 + 5 \times 1 = 6$, $n^3 + 5n$ is not prime for $n = 1$ For all other n , $n^3 + 5n = n(n^2 + 5)$ is not prime as it is the product of two other numbers not equal to 1. Hence never true.	A1	2.4
		(2)	

(4 marks)

Notes:

Condone the use of n for e.g. k for both marks.

Way 1 (Factorisation):

M1: Attempts to factorise $n^3 + 5n = n(n^2 + 5)$
Allow $n^3 + 5n = n(n^2 + \dots)$ or $n^3 + 5n = n(\dots + 5)$

A1: Requires:

- Correct factorisation $n^3 + 5n = n(n^2 + 5)$
- Substitution of $n = 1$: $1^3 + 5 \times 1 = 6$
or states $n^3 + 5n \neq 2$ oe e.g. $n^3 + 5n > 2$ oe
- “Never true”

Way 2 (Odd/Even):

M1: Attempts to substitute $n = 2k$ and either $n = 2k + 1$ or $n = 2k - 1$ oe

A1: Requires:

- Correctly factorising both even and odd forms
e.g. $8k^3 + 10k = 2k(4k^2 + 5)$ and
e.g. $(2k + 1)^3 + 5(2k + 1) = (2k + 1)((2k + 1)^2 + 5)$ or e.g. $2(2k + 1)(2k^2 + 2k + 3)$
(in this part both cases are needed)
- Substitution of $n = 1$: $1^3 + 5 \times 1 = 6$, or e.g. $k = 1$ for $n = 2k - 1$ or $k = 0$ for $n = 2k + 1$
or states $n^3 + 5n \neq 2$ oe e.g. $n^3 + 5n > 2$ oe
e.g. $2k(4k^2 + 5)$ and $2(2k + 1)(2k^2 + 2k + 3)$ are > 2 or $\neq 2$
- “Never true”

Way 3 (Odd/Even via logic):

M1: Considers $n^3 + 5n$ with “odds” and “evens” e.g.
If n is **odd** then $n^3 + 5n = \text{odd} + \text{odd} = \text{even}$
or e.g. $n^3 + 5n = n(n^2 + 5) = \text{odd}(\text{odd} + \text{odd}) = \text{odd} \times \text{even} = \text{even}$
If n is **even** then $n^3 + 5n = \text{even} + \text{even} = \text{even}$
or e.g. $n^3 + 5n = n(n^2 + 5) = \text{even}(\text{even} + \text{odd}) = \text{even} \times \text{odd} = \text{even}$

A1: Requires:

- Fully correct argument for both odds and evens
- A full justification of any assumed results e.g. n odd $\Rightarrow n^3$ is odd via algebra or e.g. n odd means $n^3 = \text{odd} \times \text{odd} \times \text{odd} = \text{odd}$
- When $n = 1$, $n^3 + 5n = 6$ so $n^3 + 5n \neq 2$
- “Never true”