13. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The growth of a particular tree is monitored over a period of time.

The height, h metres, of this tree, t years after it was planted, is modelled by the equation

$$h = 31 - Ae^{-kt}$$

where A and k are positive constants.

Given that

- exactly 10 years after it was planted, the height of the tree was 6 m
- exactly 20 years after it was planted, the height of the tree was 11 m

(a) find a complete equation for h in terms of t, giving the value of each of A and k to 3 significant figures.

(4)

Use the equation of the model to answer parts (b), (c) and (d).

According to the model, there is a limit to the height to which this tree can grow.

(b) Deduce this limit.

(1)

(c) (i) Find the initial height of the tree.

(ii) Hence explain whether this is a suitable model for the early growth of the tree.

(2)

(d) (i) Find $\frac{\mathrm{d}h}{\mathrm{d}t}$, giving your answer in simplest form.

(2)

(ii) Hence find the value of t for which the height of the tree is increasing at a rate of 30 cm per year.

(3)