

Question Number	Scheme	Marks	AOs
14	Attempts $n^2 + 2$ with $n = 3k + 1$ or $n = 3k + 2$ o.e. Look for e.g. $(3k + 1)^2 + 2 = \dots$	M1	3.1a
	e.g. $(3k + 1)^2 + 2 = 9k^2 + 6k + 3 = 3(3k^2 + 2k + 1)$ which is a multiple of 3.	A1	2.4
	Attempts $n^2 + 2$ with $n = 3k + 1$ and $n = 3k + 2$ o.e	dM1	2.1
	e.g. both $(3k + 1)^2 + 2 = 9k^2 + 6k + 3 = 3(3k^2 + 2k + 1)$ which is a multiple of 3 and $(3k + 2)^2 + 2 = 9k^2 + 12k + 6 = 3(3k^2 + 4k + 2)$ which is a multiple of 3 and e.g. Hence proven	A1	2.4
		(4)	
(4 marks)			
Notes:			

Note: If using $n = 3n + 1$ etc. then allow maximum 1110.

Ignore any attempts at $n = 3k$ throughout.

Ignore any spurious setting = 0 throughout.

M1: Starts to solve the problem by setting $n = 3k + 1$ or $n = 3k + 2$ o.e such as $n = 3k - 1$ and attempts $n^2 + 2$
Do not be concerned about their attempt to expand the brackets.
If using $n = 6k + 1$ etc. then score for one valid attempt.

A1: Achieves $(3k + 1)^2 + 2 = 9k^2 + 6k + 3 = 3(3k^2 + 2k + 1)$ **or** $(3k + 2)^2 + 2 = 9k^2 + 12k + 6 = 3(3k^2 + 4k + 2)$
or $(3k - 1)^2 + 2 = 9k^2 - 6k + 3 = 3(3k^2 - 2k + 1)$ **or** $(3k - 2)^2 + 2 = 9k^2 - 12k + 6 = 3(3k^2 - 4k + 2)$
and concludes that it is a multiple of 3 (or e.g. divisible by 3).

Condone e.g. “ $n^2 + 2$ can be divided by 3” for this mark only.

To show it is a multiple of 3 they can either factorise or comment on each term individually.

You may also see e.g. $\frac{9k^2 + 6k + 3}{3} = 3k^2 + 2k + 1$ which is acceptable.

Note that the conclusion may be seen in an overall conclusion at the end.

dM1: Attempts two cases that cover all $n \in \mathbb{N}$ that are not multiples of 3.

So, sets $n = 3k + 1$ **and** $n = 3k + 2$ o.e. such as $n = 3k - 1$ and attempts $n^2 + 2$

Do not be concerned about their attempt to expand the brackets.

If using $n = 6k + 1$ then they require 4 valid attempts that cover all $n \in \mathbb{N}$ that are not multiples of 3:

e.g. $n = 6k + 1$, $n = 6k + 2$, $n = 6k + 4$, $n = 6k + 5$

A1: Requires

- **both** $(3k+1)^2 + 2 = 9k^2 + 6k + 3$ **and** $(3k+2)^2 + 2 = 9k^2 + 12k + 6$ o.e. correct
- sufficient reasons to conclude that **both** expressions are multiples of 3
 - e.g. $9k^2 + 6k + 3 = 3(3k^2 + 2k + 1)$ or explains that each term is a multiple of 3
 - **and** e.g. $9k^2 + 12k + 6 = 3(3k^2 + 4k + 2)$ or explains that each term is a multiple of 3
- a minimal conclusion e.g. “hence proven” or “{when n is **not** a multiple of 3,} $n^2 + 2$ is always a multiple of 3” Condone the absence of “when n is **not** a multiple of 3”. Do not condone e.g. “ $n^2 + 2$ can be divided by 3” for this mark.

Alt:

M1: Starts to solve the problem by stating that if n is not a multiple of 3 then one of $(n+1)$ or $(n-1)$ must be a multiple of 3

A1: Deduces that $(n+1)(n-1)$ must be a multiple of 3

dM1: Connects the two expressions $(n+1)(n-1) + 3 \equiv n^2 + 2$

A1: Explains that as $(n+1)(n-1)$ must be a multiple of 3, $(n+1)(n-1) + 3$ must also be a multiple of 3, followed by a minimal conclusion, e.g. “hence $n^2 + 2$ is always a multiple of 3”.