

Question	Scheme	Marks	AOs
1	$\frac{1}{(r+1)(r+3)} \equiv \frac{A}{(r+1)} + \frac{B}{(r+3)} \Rightarrow A = \dots, B = \dots$	M1	3.1a
	$\sum_{r=1}^n \frac{1}{(r+1)(r+3)} =$ $\frac{1}{2 \times 2} - \frac{1}{2 \times 4} + \frac{1}{2 \times 3} - \frac{1}{2 \times 5} + \dots + \frac{1}{2n} - \frac{1}{2(n+2)} + \frac{1}{2(n+1)} - \frac{1}{2(n+3)}$	M1	2.1
	$= \frac{1}{4} + \frac{1}{6} - \frac{1}{2(n+2)} - \frac{1}{2(n+3)}$	A1	2.2a
	$= \frac{5(n+2)(n+3) - 6(n+3) - 6(n+2)}{12(n+2)(n+3)}$	M1	1.1b
	$= \frac{n(5n+13)}{12(n+2)(n+3)}$	A1	1.1b
	(5)		
	Alternative by induction: $n=1 \Rightarrow \frac{1}{8} = \frac{a+b}{12 \times 3 \times 4}, \quad n=2 \Rightarrow \frac{1}{8} + \frac{1}{15} = \frac{2(2a+b)}{12 \times 4 \times 5}$ $a+b=18, \quad 2a+b=23 \Rightarrow a = \dots, b = \dots$	M1	3.1a
	Assume true for $n = k$ so $\sum_{r=1}^k \frac{1}{(r+1)(r+3)} = \frac{k(5k+13)}{12(k+2)(k+3)}$		
	$\sum_{r=1}^{k+1} \frac{1}{(r+1)(r+3)} = \frac{k(5k+13)}{12(k+2)(k+3)} + \frac{1}{(k+2)(k+4)}$	M1	2.1
	$\frac{k(5k+13)}{12(k+2)(k+3)} + \frac{1}{(k+2)(k+4)} = \frac{k(5k+13)(k+4) + 12(k+3)}{12(k+2)(k+3)(k+4)}$	A1	2.2a
	$= \frac{5k^3 + 33k^2 + 52k + 12k + 36}{12(k+2)(k+3)(k+4)} = \frac{(k+1)(k+2)(5k+18)}{12(k+2)(k+3)(k+4)}$	M1	1.1b
	$= \frac{(k+1)(5(k+1)+13)}{12(k+1+2)(k+1+3)}$ So true for $n = k + 1$	A1	1.1b
	So $\sum_{r=1}^n \frac{1}{(r+1)(r+3)} = \frac{n(5n+13)}{12(n+2)(n+3)}$		
(5)			
(5 marks)			

Question 1 notes:

Main Scheme

M1: Valid attempt at partial fractions

M1: Starts the process of differences to identify the relevant fractions at the start and end

A1: Correct fractions that do not cancel

M1: Attempt common denominator

A1: Correct answer

Alternative by Induction:

M1: Uses $n = 1$ and $n = 2$ to identify values for a and b

M1: Starts the induction process by adding the $(k + 1)^{\text{th}}$ term to the sum of k terms

A1: Correct single fraction

M1: Attempt to factorise the numerator

A1: Correct answer and conclusion