

9. A company plans to build a new fairground ride. The ride will consist of a capsule that will hold the passengers and the capsule will be attached to a tall tower. The capsule is to be released from rest from a point half way up the tower and then made to oscillate in a vertical line.

The vertical displacement,  $x$  metres, of the top of the capsule below its initial position at time  $t$  seconds is modelled by the differential equation,

$$m \frac{d^2x}{dt^2} + 4 \frac{dx}{dt} + x = 200 \cos t, \quad t \geq 0$$

where  $m$  is the mass of the capsule including its passengers, in thousands of kilograms.

The maximum permissible weight for the capsule, including its passengers, is 30 000 N.

Taking the value of  $g$  to be  $10 \text{ ms}^{-2}$  and assuming the capsule is at its maximum permissible weight,

- (a) (i) explain why the value of  $m$  is 3  
(ii) show that a particular solution to the differential equation is

$$x = 40 \sin t - 20 \cos t$$

- (iii) hence find the general solution of the differential equation.

(8)

- (b) Using the model, find, to the nearest metre, the vertical distance of the top of the capsule from its initial position, 9 seconds after it is released.

(4)