

Question	Scheme	Marks	AOs
4	$\int_0^3 \frac{kx}{x^2+6} dx + \int_3^{\infty} \frac{k}{x^2-4} dx = \frac{1}{4} \Rightarrow k \left( [\dots]_0^3 + [\dots]_3^{\infty} \right) = \frac{1}{4} \Rightarrow k = \dots$	M1	3.1a
	$(k) \int_0^3 \frac{x}{x^2+6} dx = (k) \left[ \frac{1}{2} \ln(x^2+6) \right]_0^3 = (k) \left( \frac{1}{2} \ln(15) - \frac{1}{2} \ln(6) \right)$	M1 A1	1.1b 1.1b
	$(k) \int \frac{1}{x^2-4} dx = (k) \frac{1}{4} \ln \left( \frac{x-2}{x+2} \right)$	M1	1.1b
	$\int_3^{\infty} \frac{1}{x^2-4} dx = \lim_{t \rightarrow \infty} \left[ \frac{1}{4} \ln \left( \frac{x-2}{x+2} \right) \right]_3^t$ $= \frac{1}{4} \left( \lim_{t \rightarrow \infty} \ln \left( \frac{t-2}{t+2} \right) - \ln \left( \frac{1}{5} \right) \right) = \frac{1}{4} \ln 5$	M1 A1	3.1a 1.1b
	So $\frac{k}{2} \ln \left( \frac{15}{6} \right) + \frac{k}{4} \ln 5 = \frac{1}{4} \Rightarrow k \ln \left( \frac{5}{2} \right)^2 + k \ln(5) = 1$ so $k = \frac{1}{\ln \left( \frac{125}{4} \right)}$ .	M1 A1	2.1 1.1b
		(8)	

(8 marks)

**Notes:**

**M1:** For a complete overall method. Correct expression for the total area formed, a sum of two areas with an attempt made for both areas in terms of  $k$  and with area set equal to  $\frac{1}{4}$ .

**M1:** Correct form for integral of left hand part (with or without  $k$ ) and attempts to apply the limits. Accept  $a \ln(x^2+6)$ .

**A1:** Integral correct, with limits applied correctly. No need to combine lns at this stage.

**M1:** Correct form integral for the right hand part (with or without  $k$ ), quoted directly or uses partial fractions. Accept  $A \ln \left( \frac{x-2}{x+2} \right)$  or  $A \ln(x-2) - B \ln(x+2)$  from an attempt at partial fractions.

**M1:** Applies the limits on the improper integral, 3 as lower limit and  $t$  as upper with  $t \rightarrow \infty$  to obtain a value for the integral. If partial fractions used, the log terms will need combining first.

**A1:** Fully correct integral for right hand section, as in scheme or equivalent, with or without  $k$ .

**M1:** Adds the results of both the integrals and equates to  $\frac{1}{4}$ , including  $k$ , and uses correct log laws to combine their log terms to find  $k$ .

**A1:** Correct answer. Accept any equivalents in the correct form, e.g.  $\frac{1}{\ln(31.25)}$  or statement

that  $a = \frac{125}{4}$  etc.