Question	Scheme	Marks	AOs
7(i)	When $n = 1$, LHS $= \begin{pmatrix} 4 & -1 \\ 9 & -2 \end{pmatrix}$, RHS $= \begin{pmatrix} 3\times1+1 & -1 \\ 9\times1 & 1-3\times1 \end{pmatrix} = \begin{pmatrix} 4 & -1 \\ 9 & -2 \end{pmatrix}$. So the statement is true for $n = 1$	B1	2.2a
	Assume true for $n = k$, so $\begin{pmatrix} 4 & -1 \\ 9 & -2 \end{pmatrix}^k = \begin{pmatrix} 3k+1 & -k \\ 9k & 1-3k \end{pmatrix}$	M1	2.4
	Then $\begin{pmatrix} 4 & -1 \\ 9 & -2 \end{pmatrix}^{k+1} = \begin{pmatrix} 4 & -1 \\ 9 & -2 \end{pmatrix}\begin{pmatrix} 3k+1 & -k \\ 9k & 1-3k \end{pmatrix}$ or $\begin{pmatrix} 3k+1 & -k \\ 9k & 1-3k \end{pmatrix}\begin{pmatrix} 4 & -1 \\ 9 & -2 \end{pmatrix}$	M1	2.1
	$= \begin{pmatrix} 4(3k+1)-9k & -4k-(1-3k) \\ 9(3k+1)-18k & -9k-2(1-3k) \end{pmatrix}$ or $\begin{pmatrix} 4(3k+1)-9k & -(3k+1)+2k \\ 36k+9(1-3k) & -9k-2(1-3k) \end{pmatrix}$	A1	1.1b
	$= \begin{pmatrix} 3(k+1)+1 & -(k+1) \\ 9(k+1) & 1-3(k+1) \end{pmatrix}$	A1	1.1b
	Hence the result is true for $n = k+1$. Since it is <u>true for $n = 1$</u>, and <u>if true for $k = n$ then true for $n = k+1$</u>, thus by mathematical induction the <u>result holds for all n</u> $\in$ □	A1 cso	2.4
		(6)	
(ii)	(a) $2^2 = 4 \not< 4 = 2^2$ OR $3^2 = 9 \not< 8 = 2^3$ OR $4^2 = 16 \not< 16 = 2^4$	B1	1.1b
	(b) The statement $2k+1 < k^2$ is not true for all positive integers.	B1	1.1b
	(c) The statement in line 4 is true for positive integers $k > 2$ so the induction hypothesis is true for $n > 2$. So the induction holds from any base case greater than 2.	M1	2.3
	Since the result is true for $n = 5$ as $5^2 = 25 < 32 = 2^5$ and $2k+1 < k^2$ also true for $k > 5$ so the induction holds with base case $n = 5$.	A1	2.4
	But not true for $n = 2$, 3 or 4 as $2^2 = 4 \not< 4 = 2^2$ and $3^2 = 9 \not< 8 = 2^3$ and $4^2 = 16 \not< 16 = 2^4$. Hence true for $n = 1$ and for n …5	A1	2.1
		(5)	
			(11 marks)

Notes:

(a)

B1: Shows the general form holds for $n = 1$.

M1: Makes the inductive assumption, assume true for $n = k$.

M1: Attempts the multiplication either way.

A1: Correct matrix in terms of k.

A1: Rearranged into correct form to show true for $k + 1$.

A1: Completes the inductive argument conveying **all** three underlined points or equivalent at some point in their argument.

(b)(i)

B1: Provides a suitable counter example using $n = 2$, 3 or 4. Accept $=$ in place of $\nless$ as long as there is a suitable conclusion with it.

(b)(ii)

B1: Identifies the error as in the scheme or equivalent (e.g. $k^2 + 2k + 1 < 2k^2$ is not always true).

(b)(iii)

M1: Identifies that the induction is valid as long as $2k+1 < k^2$ is true which happens for $k..3$ (accept any value greater than 3 for this mark).

A1: Correct base case of 5 and explains the proof given holds for integers greater than or equal to 5.

A1: Complete argument correct. All positive integers satisfying the inequality identified, with demonstration that 2, 3 and 4 do not.