| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **8(a)(i)** | Container contains $3+0.25t-0.125t = 3+0.125t$ litres after $t$ minutes | B1 | 3.3 |
| **(ii)** | Rate of contaminant out $= 0.125\times\dfrac{m}{3+0.125t}$ mg per minute | M1 | 3.3 |
| | Rate of contaminant in $= 0.25\times(5-\mathrm{e}^{-0.1t})$ mg per minute | B1 | 2.2a |
| | $\dfrac{\mathrm{d}m}{\mathrm{d}t}=\dfrac{5-\mathrm{e}^{-0.1t}}{4}-\dfrac{m}{24+t}$* | A1* | 1.1b |
| | | **(4)** | |
| **(b)** | Rearranges to form $\dfrac{\mathrm{d}m}{\mathrm{d}t}+\dfrac{m}{24+t}=\dfrac{5-\mathrm{e}^{-0.1t}}{4}$ and attempts integrating factor (may be by recognition). | M1 | 3.1a |
| | I.F. $=\left(\mathrm{e}^{\int\frac{1}{24+t}\,\mathrm{d}t}=\mathrm{e}^{\ln(24+t)}\right)=24+t$ | A1 | 1.1b |
| | $(24+t)m=\dfrac{1}{4}\int(24+t)(5-\mathrm{e}^{-0.1t})\,\mathrm{d}t=\dfrac{1}{4}\int 120+5t-24\mathrm{e}^{-0.1t}-t\mathrm{e}^{-0.1t}\,\mathrm{d}t=..$ | M1 | 3.1a |
| | $=\dfrac{1}{4}\left(120t+\dfrac{5t^2}{2}-\dfrac{24\mathrm{e}^{-0.1t}}{-0.1}+\ldots\right)$ | A1 | 1.1b |
| | $\int t\mathrm{e}^{-0.1t}\,\mathrm{d}t=t\dfrac{\mathrm{e}^{-0.1t}}{-0.1}-\int 1\times\dfrac{\mathrm{e}^{-0.1t}}{-0.1}\,\mathrm{d}t=t\dfrac{\mathrm{e}^{-0.1t}}{-0.1}-\dfrac{\mathrm{e}^{-0.1t}}{(-0.1)^2}$ | M1<br>A1 | 1.1b<br>1.1b |
| | So $(24+t)m=\dfrac{5}{8}t^2+30t+85\mathrm{e}^{-0.1t}+\dfrac{5}{2}t\mathrm{e}^{-0.1t}+c$ | | |
| | When $t=0$, $m=0$ as initially no contaminant in the container, so $0=0+0+85+0+c\Rightarrow c=-85$ | M1 | 3.4 |
| | $m=\dfrac{1}{24+t}\left(\dfrac{5}{8}t^2+30t+85\mathrm{e}^{-0.1t}+\dfrac{5}{2}t\mathrm{e}^{-0.1t}-85\right)$ | A1 | 2.2b |
| | | **(8)** | |
| **(c)** | When $t=30$ $m=25.65677...$ and $V=6.75$, hence the concentration is 3.80 mg per litre. | M1 | 3.4 |
| | This resembles the measured value very closely and could easily be explained by minor inaccuracies in measurements, so the model seems to be suitable over this timeframe. | A1 | 3.5a |
| | | **(2)** | |
| | | | **(14 marks)** |

**Notes:**

**(a)(i)**

**B1:** A correct expression for the volume, may be unsimplified.

**(ii)**

**M1:** Expresses the amount of contaminant out in terms of $m$ and $t$.

**B1:** Correct interpretation for amount of contaminant entering the container.

**A1*:** Puts all the components together to form the correct differential equation.

**(b)**

**M1:** Identifies the problem as a first order linear problem requiring integrating factor (by finding it or by recognition.

**A1:** Correct integrating factor

**M1:** Multiplies through by the IF, expands brackets on RHS and attempts the integration.

**A1:** Correct integration for first three terms.

**M1:** Integration by parts used on the $te^{-0\,1t}$ term.

**A1**: Correct integration by parts.

**M1:** Uses the initial conditions to find the constant of integration – must have a constant of integration for this mark to be awarded.

**A1**: Correct expression for $m$, need not be simplified.

**(c)**

**M1:** Calculates the concentration from the model at $t = 30$

**A1**: Correct concentration found and uses it to make a comment on the validity of the model.