Question	Scheme	Marks	AOs
5(a)	The tank initially contains 100L. 3 L are entering every minute and 2 L are leaving every minute so overall 1 L increase in volume each minute so the tank contains $100+t$ litres after t minutes	M1	3.3
	2 L leave the tank each minute and if there are Sg of salt in the tank, the concentration will be $\frac{S}{100+t} g/L$ so salt leaves the tank at a rate of $2\times\frac{S}{100+t} g$ per minute	M1	3.3
	Salt enters the tank at a rate of $3\times 1g$ per minute	B1	2.2a
	$\therefore \frac{\mathrm{d}S}{\mathrm{d}t}=3-\frac{2S}{100+t}$* cso	A1*	1.1b
		(4)	
(b)	$\frac{\mathrm{d}S}{\mathrm{d}t}+\frac{2S}{100+t}=3$		
	$I=\mathrm{e}^{\int\frac{2}{100+t}\mathrm{d}t}=(100+t)^2 \Rightarrow S(100+t)^2=\int 3(100+t)^2\mathrm{d}t$	M1	3.1b
	$S(100+t)^2=(100+t)^3(+c)$ OR $S(100+t)^2=30\,000t+300t^2+t^3(+c)$	A1	1.1b
	$t=0,\ S=0\Rightarrow c=-10^6$	M1	3.4
	$t=10\Rightarrow S=100+10-\frac{10^6}{(100+10)^2}$	dM1	1.1b

	OR $S(100+10)^2=(100+10)^3(+c)\Rightarrow S=...$		
	$=$ awrt 27 (g) or $\frac{3310}{121}$(g)	A1	2.2b
		(5)	
(c)	Concentration is $\left(100+t-\frac{10^6}{(100+t)^2}\right)\div(100+t)=0.9$ OR $S=0.9\ 100+t\ \Rightarrow 0.9\ 100+t\ =\ 100+t\ -\frac{10^6}{100+10^{\ 2}}$ OR $S=0.9\ 100+t\ \Rightarrow 0.9\ 100+t\ ^3=\ 100+t\ ^3-10^6$	M1	3.4
	$(100+t)^3=10^7\Rightarrow t=...$ OR $t^3+300t^2+30\,000t-9\,000\,000=0\Rightarrow t=...$	dM1	1.1b
	$t=$ awrt 115 (minutes)	A1	2.2b
		(3)	
(d)	E.g. • It is unlikely that mixing is instantaneous • The model will only be valid when the tank is not full • When the valve is closed, the model is not valid • It is unlikely that the concentration of salt water entering the tank remains exactly the same	B1	3.5a
		(1)	
			(13 marks)

Notes

(a)

M1: A suitable explanation for the "$100 + t$" e.g. as a minimum (v) = $100 + 3t - 2t = 100 + t$

M1: A suitable explanation for the $\frac{2S}{100+t}$

There need to be some explanation (words) for this part of the formula.

e.g. the concentration of (salt) $=\frac{S}{100+t}$ therefore (salt) out $=2\times\frac{S}{100+t}=\frac{2S}{100+t}$

e.g. salt out = $\frac{2S}{\text{volume of water}}=\frac{2S}{100+t}$

Note: M0 for $2\times\frac{S}{100+t}=\frac{2S}{100+t}$ only with no explanation

B1: Correct interpretation for the "3" e.g. salt in = 3 or $\frac{\mathrm{d}S}{\mathrm{d}t}$ in = 3

Note: Salt **water** in = 3 is **B0**

A1*: Puts all the components together to form the given differential equation cso

(b)

M1: Uses the model to find the integrating factor and attempts the solution of the differential equation. Look for $I.F. = e^{\int \frac{2}{100+t}\,dt} \Rightarrow S \times \text{'their } I.F.\text{'} = \int 3 \times \text{'their } I.F.\text{'}\,dt$

A1: Correct solution condone missing $+ c$

For the next three mark there must be a constant of integration

M1: Interprets the initial conditions, $t = 0 \;\; S = 0$, and uses in their equation to find the constant of integration.

dM1: Dependent on having a constant of integration. Uses their solution to the problem to find the amount of salt after 10 minutes.

A1: Awrt 27 or $\frac{3310}{121}$. (If the units are stated they must be correct)

Note: If achieves $S(100+t)^2 = 30\,000t + 300t^2 + t^3 + c$ the constant of integration $c = 0$ and the correct amount of salt can be achieved. If there is no $+ c$ the maximum they can score is M1A1M0M0A0

Notes continued

(c)

Note: Look out for setting $S = 0.9$ in this part, which scores no marks.

M1: Uses their solution to the model and divides by $100 + t$ as an interpretation of the concentration and sets $= 0.9$.

Alternatively recognises that the amount of salt $= 0.9(100 + t)$ and substitutes for S in their solution to the model.

dM1: Dependent on previous method mark. Solves their equation to obtain a value for t. May use a calculator.

A1: Awrt 115 (If the units are stated they must be correct) or 1hr 45 mins with units

(d)

B1: Evaluates the model by making a suitable comment – see scheme for examples.