Question	Scheme	Marks	AOs
6	**<u>Way 1</u>** $f(k+1)-f(k)$		
	When $n=1$, $3^{2n+4}-2^{2n}=729-4=725$ $(725=145\times5)$ so the statement is true for $n=1$	B1	2.2a
	Assume true for $n=k$ so $3^{2k+4}-2^{2k}$ is divisible by 5	M1	2.4
	$f(k+1)-f(k)=3^{2k+6}-2^{2k+2}-3^{2k+4}+2^{2k}$	M1	2.1
	either $8f(k)+5\times2^{2k}$ or $3f(k)+5\times3^{2k+4}$	A1	1.1b
	$f(k+1)=9f(k)+5\times2^{2k}$ or $f(k+1)=4f(k)+5\times3^{2k+4}$ o.e.	A1	1.1b
	<u>If true for $n=k$ then it is true for</u>	A1	2.4

	$n = k + 1$ and as it is true for $n = 1$, the statement is true for all (positive integers) n. (Allow 'for all values')		
		(6)	
	Way 2 $\mathrm{f}(k+1)$		
	When $n = 1$, $3^{2n+4} - 2^{2n} = 729 - 4 = 725$ $(725 = 145\times 5)$ so the statement is true for $n = 1$	B1	2.2a
	Assume true for $n = k$ so $3^{2k+4} - 2^{2k}$ is divisible by 5	M1	2.4
	$\mathrm{f}\left(k+1\right) = 3^{2(k+1)+4} - 2^{2(k+1)}\left(= 3^{2k+6} - 2^{2k+2}\right)$	M1	2.1
	$\mathrm{f}\ \ k+1\ = 9\mathrm{f}\ \ k\ + 5\times 2^{2k}$ or $\mathrm{f}\ \ k+1\ = 4\mathrm{f}\ \ k\ + 5\times 3^{2k+4}$ o.e.	A1 A1	1.1b 1.1b
	If true for $n = k$ then it is true for $n = k + 1$ and as it is true for $n = 1$, the statement is true for all (positive integers) n. (Allow 'for all values')	A1	2.4
		(6)	
	Way 3 $\mathrm{f}(k) = 5M$		
	When $n = 1$, $3^{2n+4} - 2^{2n} = 729 - 4 = 725$ $(725 = 145\times 5)$ so the statement is true for $n = 1$	B1	2.2a
	Assume true for $n = k$ so $3^{2k+4} - 2^{2k} = 5M$	M1	2.4
	$\mathrm{f}\left(k+1\right) = 3^{2(k+1)+4} - 2^{2(k+1)}\left(= 3^{2k+6} - 2^{2k+2}\right)$	M1	2.1
	$\left(\mathrm{f}\left(k+1\right) = 3^2\times 3^{2k+4} - 2^2\times 2^{2k} = 3^2\times\left(5M + 2^{2k+2}\right) - 2^2\times 2^{2k}\right)$ $\mathrm{f}\ \ k+1\ = 45M + 5\times 2^{2k}$ o.e. OR $\left(\mathrm{f}\left(k+1\right) = 3^2\times 3^{2k+4} - 2^2\times 2^{2k} = 3^2\times 3^{2k+4} - 2^2\times\left(3^{2k+4} - 5M\right)\right)$ $\mathrm{f}\ \ k+1\ = 5\times 3^{2k+4} + 20M$ o.e.	A1 A1	1.1b 1.1b
	If true for $n = k$ then it is true for $n = k + 1$ and as it is true for $n = 1$, the statement is true for all (positive integers) n. (Allow 'for all values')	A1	2.4
		(6)	
	Way 4 $\mathrm{f}(k+1) + \mathrm{f}(k)$		
	When $n = 1$, $3^{2n+4} - 2^{2n} = 729 - 4 = 725$ $(725 = 145\times 5)$ so the statement is true for $n = 1$	B1	2.2a
	Assume true for $n = k$ so $3^{2k+4} - 2^{2k}$ is divisible by 5	M1	2.4
	$\mathrm{f}\left(k+1\right) + \mathrm{f}\left(k\right) = 3^{2k+6} - 2^{2k+2} + 3^{2k+4} - 2^{2k}$	M1	2.1
	$\mathrm{f}\left(k+1\right) + \mathrm{f}\left(k\right) = 3^2\times 3^{2k+4} - 2^2\times 2^{2k} + 3^{2k+4} - 2^{2k}$	A1	1.1b

	leading to $10\times3^{2k+4}-5\times2^{2k}$		
	$\text{f}\ \ k+1\ =5\left[2\times3^{2k+4}-2^{2k}\right]-\text{f}(k)$ o.e.	A1	1.1b
	<u>If true for $n = k$ then it is true for</u> <u>$n = k + 1$</u> and as it is <u>true for $n = 1$</u>, the statement is <u>true for all</u> <u>(positive integers) n</u>. (Allow 'for all values')	A1	2.4
		(6)	
	<u>Way 5</u> f(k +1) – 'M'f(k) (Selecting a value of M that will lead to multiples of 5)		
	When $n = 1$, $3^{2n+4}-2^{2n}=729-4=725$ ($725=145\times5$)so the statement is true for $n = 1$	B1	2.2a
	Assume true for $n = k$ so $3^{2k+4}-2^{2k}$ is divisible by 5	M1	2.4
	$\text{f}\left(k+1\right)-\text{'M'f}\left(k\right)=3^{2k+6}-2^{2k+2}-\text{'M'}\times3^{2k+4}+\text{'M'}\times2^{2k}$	M1	2.1
	$\text{f}\ \ k+1\ -\text{'M'f}\ \ k\ \ =\ 9-\text{'M'}\ \times3^{2k+4}-\ \ 4-\text{'M'}\ \times2^{2k}$	A1	1.1b
	$\text{f}\ \ k+1\ =\ 9-\text{'M'}\ \times3^{2k+4}-\ \ 4-\text{'M'}\ \times2^{2k}+\text{'M'f}\ \ k$ o.e.	A1	1.1b
	<u>If true for $n = k$ then it is true for</u> <u>$n = k + 1$</u> and as it is <u>true for $n = 1$</u>, the statement is <u>true for all</u> <u>(positive integers) n</u>. (Allow 'for all values')	A1	2.4
		(6)	

(6 marks)

Notes

<u>Way 1</u> f(k +1) – f(k)

B1: Shows the statement is true for $n = 1$. Needs to show f(1) = 725 and conclusion true for $n = 1$, this statement can be recovered in their conclusion if says e.g. true for $n = 1$

M1: Makes an assumption statement that assumes the result is true for $n = k$. Assume (true for) $n=k$ is sufficient. This mark may be recovered in their conclusion if they say e.g. if true for $n=k$ then …etc

M1: Attempts f(k +1) – f(k) or equivalent work

A1: Achieves a correct simplified expression for f(k +1) – f(k)

A1: Achieves a correct expression for f(k +1) in terms of f(k)

A1: Correct complete conclusion. This mark is dependent on **all** previous marks. It is gained by conveying the ideas of **all** underlined points either at the end of their solution **or** as a narrative in their solution.

<u>Way 2</u> f(k +1)

B1: Shows the statement is true for $n = 1$. Needs to show f(1) = 725 and conclusion true for $n = 1$, this statement can be recovered in their conclusion if says e.g. true for $n = 1$.

M1: Makes an assumption statement that assumes the result is true for $n = k$. Assume (true for) $n=k$ is sufficient. This mark may be recovered in their conclusion if they say e.g. if true for $n=k$ then …etc

M1: Attempts f(k +1)

A1: Correctly achieves either $9\text{f}\ \ k$ **or** 5×2^{2k} or either $4\text{f}\ \ k$ **or** $5\times3^{2k+4}$

A1: Achieves a correct expression for f(k +1) in terms of f(k)
A1: Correct complete conclusion. This mark is dependent on **all** previous marks. It is gained by conveying the ideas of **all** underlined points either at the end of their solution **or** as a narrative in their solution.

Way 3 f(k) = 5M
B1: Shows the statement is true for n = 1. Needs to show f(1) = 725 and conclusion true for n = 1, this statement can be recovered in their conclusion if says e.g. true for n = 1.
M1: Makes an assumption statement that assumes the result is true for $n = k$. Assume (true for) $n = k$ is sufficient. This mark may be recovered in their conclusion if they say e.g. if true for $n = k$ then …etc
M1: Attempts f(k +1)
A1: Correctly achieves either $45M$ **or** 5×2^{2k} or either $20M$ **or** $5\times 3^{2k+4}$
A1: Achieves a correct expression for f(k +1) in terms of M and 2^{2k} or M and 3^{2k+4}
A1: Correct complete conclusion. This mark is dependent on **all** previous marks. It is gained by conveying the ideas of **all** underlined points either at the end of their solution **or** as a narrative in their solution.

Way 4 f(k +1) + f(k)
B1: Shows the statement is true for n = 1. Needs to show f(1) = 725 and conclusion true for n = 1, this statement can be recovered in their conclusion if says e.g. true for n = 1
M1: Makes an assumption statement that assumes the result is true for $n = k$. Assume (true for) $n = k$ is sufficient. This mark may be recovered in their conclusion if they say e.g. if true for $n = k$ then …etc
M1: Attempts f(k +1) + f(k) or equivalent work
A1: Achieves a correct simplified expression for f(k +1) + f(k)
A1: Achieves a correct expression for $\mathrm{f}(k+1) = 5\left[2\times 3^{2k+4} - 2^{2k}\right] - \mathrm{f}(k)$
A1: Correct complete conclusion. This mark is dependent on **all** previous marks. It is gained by conveying the ideas of **all** underlined points either at the end of their solution **or** as a narrative in their solution.

Notes continued

Way 5 f(k +1) – Mf(k) (Selects a suitable value for M which leads to divisibility of 5)
B1: Shows the statement is true for n = 1. Needs to show f(1) = 725 and conclusion true for n = 1, this statement can be recovered in their conclusion if says e.g. true for n = 1
M1: Makes an assumption statement that assumes the result is true for $n = k$. Assume (true for) $n = k$ is sufficient. This mark may be recovered in their conclusion if they say e.g. if true for $n = k$ then …etc
M1: Attempts f(k +1) – Mf(k) or equivalent work
A1: Achieves a correct simplified expression, $\mathrm{f}(k+1) - \text{'M'}\mathrm{f}(k)$ which is divisible by 5

$$\mathrm{f}(k+1) - \text{'M'}\mathrm{f}(k) = (9 - \text{'M'})\times 3^{2k+4} - (4 - \text{'M'})\times 2^{2k}$$

A1: Achieves a correct expression for $\mathrm{f}(k+1) = (9 - \text{'M'})\times 3^{2k+4} - (4 - \text{'M'})\times 2^{2k} + \text{'M'}\mathrm{f}(k)$ which is divisible by 5
A1: Correct complete conclusion. This mark is dependent on **all** previous marks. It is gained by conveying the ideas of **all** underlined points either at the end of their solution **or** as a narrative in their solution.