Question	Scheme	Marks	AOs
7(a) Way 1	$1+2\lambda=1+t$ $-1-\lambda=-t$ $4+3\lambda=3+2t$ $\Rightarrow t=...\text{or }\lambda=...$	M1	3.1a
	Checks the third equation with $t=2$ and $\lambda=1$ Or shows that the coordinate (3, -2, 7) lies on both lines	A1	1.1b
	As the lines intersect at a point the lines lie in the same plane.	A1	2.4
		(3)	
(a) Way 2	$1=1+2\lambda+t$ $-1=\ \ -\lambda-t$ $4=3+3\lambda+2t$ $\Rightarrow t=...\text{or }\lambda=...$	$1=1+2\lambda+t$ $0=-1-\lambda-t$ $3=4+3\lambda+2t$ $\Rightarrow t=...\text{or }\lambda=...$	M1	3.1a
	Checks the third equation with $t=2$ and $\lambda=-1$	Checks the third equation with $t=-2$ and $\lambda=1$	A1	1.1b
	Second coordinates lie on the plane; therefore, the lines lie on the same plane	A1	2.4
		(3)	
(a) Way 3	$x=1+t,\quad y=-t,\quad z=3+2t$ $\frac{1+t-1}{2}=\frac{-t+1}{-1}=\frac{3+2t-4}{3}$ Solves a pair of equations $t=...$	M1	3.1a
	Solve two pairs of equations to find $t=2$	A1	1.1b
	As the lines intersect at a point the lines lie in the same plane.	A1	2.4
		(3)	
(a) Way 4 (Using Further Pure 2 knowledge)	$\begin{pmatrix}2\\-1\\3\end{pmatrix}.\begin{pmatrix}x\\y\\z\end{pmatrix}\Rightarrow 2x-y+3z=0$ and $\begin{pmatrix}1\\-1\\2\end{pmatrix}.\begin{pmatrix}x\\y\\z\end{pmatrix}\Rightarrow x-y+2z=0$ attempts to solve the equations to find a normal vector OR attempts the cross product $\begin{pmatrix}2\\-1\\3\end{pmatrix}\times\begin{pmatrix}1\\-1\\2\end{pmatrix}=...$ **AND** either finds the equation of one plane **OR** finds dot product between the normal and one coordinate	M1	3.1a

	$\mathbf{r}.\begin{pmatrix}1\\-1\\-1\end{pmatrix}=\begin{pmatrix}1\\-1\\4\end{pmatrix}.\begin{pmatrix}1\\-1\\-1\end{pmatrix}=\ldots$ or $\mathbf{r}.\begin{pmatrix}1\\-1\\-1\end{pmatrix}=\begin{pmatrix}1\\0\\3\end{pmatrix}.\begin{pmatrix}1\\-1\\-1\end{pmatrix}=\ldots$ **OR** $\begin{pmatrix}1\\-1\\4\end{pmatrix}.\begin{pmatrix}1\\-1\\-1\end{pmatrix}=\ldots$ or $\begin{pmatrix}1\\0\\3\end{pmatrix}.\begin{pmatrix}1\\-1\\-1\end{pmatrix}=\ldots$		
	Achieves the correct planes containing **each** line $\mathbf{r}.\begin{pmatrix}1\\-1\\-1\end{pmatrix}=-2$ or $x-y-z=-2$ o.e. **OR** Shows that $\begin{pmatrix}1\\-1\\4\end{pmatrix}.\begin{pmatrix}1\\-1\\-1\end{pmatrix}=-2$ **and** $\begin{pmatrix}1\\0\\3\end{pmatrix}.\begin{pmatrix}1\\-1\\-1\end{pmatrix}=-2$ o.e.	A1	1.1b
	Both planes are the same, therefore the lines lie in the same plane.	A1	2.4
		(3)	
(b)	e.g. $\mathbf{r}=\begin{pmatrix}1\\0\\3\end{pmatrix}+p\begin{pmatrix}2\\-1\\3\end{pmatrix}+q\begin{pmatrix}1\\-1\\2\end{pmatrix}$ or $\mathbf{r}=\begin{pmatrix}1\\-1\\4\end{pmatrix}+p\begin{pmatrix}2\\-1\\3\end{pmatrix}+q\begin{pmatrix}1\\-1\\2\end{pmatrix}$ or $\mathbf{r}=\begin{pmatrix}3\\-2\\7\end{pmatrix}+p\begin{pmatrix}2\\-1\\3\end{pmatrix}+q\begin{pmatrix}1\\-1\\2\end{pmatrix}$ or $\mathbf{r}=\begin{pmatrix}3\\-2\\7\end{pmatrix}+p\begin{pmatrix}0\\-1\\1\end{pmatrix}+q\begin{pmatrix}1\\-1\\2\end{pmatrix}$ or $\mathbf{r}.k\begin{pmatrix}1\\-1\\-1\end{pmatrix}=-2k$	B1	2.5
		(1)	
(c) **Way 1**	$\begin{pmatrix}2\\-1\\3\end{pmatrix}\bullet\begin{pmatrix}1\\-1\\2\end{pmatrix}=2+1+6$	M1	1.1b
	$\sqrt{2^2+(-1)^2+3^2}\sqrt{1^2+(-1)^2+2^2}\cos\theta=9$ $\Rightarrow\cos\theta=\dfrac{9}{\sqrt{2^2+(-1)^2+3^2}\sqrt{1^2+(-1)^2+2^2}}$	dM1	2.1
	$\theta=11$ cao	A1	1.1b
		(3)	

Way 2 (Using Further Pure 2 knowledge)	$\begin{pmatrix}2\\-1\\3\end{pmatrix}\times\begin{pmatrix}1\\-1\\2\end{pmatrix}=\begin{pmatrix}1\\-1\\-1\end{pmatrix}$	M1	1.1b
	$\sqrt{2^2+(-1)^2+3^2}\sqrt{1^2+(-1)^2+2^2}\sin\theta=\sqrt{1^2+(-1)^2+(-1)^2}$ $\Rightarrow \sin\theta=\dfrac{\sqrt{1^2+(-1)^2+(-1)^2}}{\sqrt{2^2+(-1)^2+3^2}\sqrt{1^2+(-1)^2+2^2}}$	dM1	2.1
	$\theta=11$ cao	A1	1.1b
		(3)	

(7 marks)

Notes

(a)

Allow using $\begin{pmatrix}1\\3\\0\end{pmatrix}$ instead of $\begin{pmatrix}1\\0\\3\end{pmatrix}$ for the method mark.

<u>Way 1</u>

M1: Starts by attempting to find where the two lines intersect. They must set up a parametric equation for line 1 (allow sign slips and as long as the intention is clear), forms simultaneous equations by equating coordinates and attempts to solve to find a value for $t=\ldots$ or $\lambda=\ldots$.

A1: Shows that there is a unique solution by checking the third equation or shows that the coordinate (3, -2, 7) lies on both lines.

A1: Achieves the correct values $t=2$ and $\lambda=1$, checks the third equation and concludes that either

- a common point,
- the lines intersect
- the equations are consistent

therefore, the lines lie in the same plane

<u>Way 2</u>

M1: Finds the vector equation of the plane with the both direction vectors and one coordinate (allow a sign slip), sets equal to the other coordinate, forms simultaneous equations and attempts to solve to find a value for $t=\ldots$ or $\lambda=\ldots$.

A1: Shows that the other coordinate lies on the plane by checking the third equation.

A1: Achieves the correct values $t=-2$ and $\lambda=1$ or $t=2$ and $\lambda=-1$ and concludes that the second coordinate lie on the plane; therefore, the lines lie on the same plane

<u>Way 3</u>

M1: Substitutes line 2 into line 1 and solves a pair of equations to find a value for *t*. Allow slip with the position of 0 and sign slips as long as the intention is clear.

A1: Solve two pairs of equations to achieve $t = 2$ for each.
A1: Achieves the correct value $t = 2$ and concludes that either

- a common point,
- the lines intersect
- the equations are consistent

therefore, the lines lie in the same plane

<u>Way 4</u> (Using Further Pure 2 knowledge)
M1: A complete method to finds a vector which is normal to both lines and attempts to finds the equation of the plane containing one line.
A1: Achieves the correct equation for the plane containing each line.
A1: Conclusion, planes are the same, therefore the lines lie in the same plane.

(b) **This may be seen in part (a)**
B1: Correct **vector** equation allow any letter for the scalers.

Must start with $\mathbf{r} = \ldots$ and uses two out of the following direction vectors $\pm\begin{pmatrix}1\\-1\\2\end{pmatrix}$, $\pm\begin{pmatrix}2\\-1\\3\end{pmatrix}$ or $\pm\begin{pmatrix}0\\-1\\1\end{pmatrix}$ and one of the following position vectors $\begin{pmatrix}1\\0\\3\end{pmatrix}$, $\begin{pmatrix}1\\-1\\4\end{pmatrix}$ or $\begin{pmatrix}3\\-2\\7\end{pmatrix}$

(c)
<u>Way 1</u>
M1: Calculates the scalar product between the direction vectors, allow one slip, if the intention is clear
dM1: Dependent on the previous method mark. Applies the scalar product formula with their scalar product to find a value for $\cos\theta$
A1: Correct answer only
<u>Way 2</u> (Using Further Pure 2 knowledge)
M1: Calculates the vector product between the direction vectors, allow one slip, if the intention is clear
dM1: Dependent on the previous method mark. Applies the vector product formula with their vector product to find a value for $\sin\theta$
A1: Correct answer only