Question	Scheme	Marks	AOs
8(a)	$\frac{\mathrm{d}^2w}{\mathrm{d}t^2}=\frac{5}{2}\left(\frac{\mathrm{d}w}{\mathrm{d}t}-\frac{\mathrm{d}s}{\mathrm{d}t}\right)$ or $\frac{\mathrm{d}s}{\mathrm{d}t}=\frac{\mathrm{d}w}{\mathrm{d}t}-\frac{2}{5}\frac{\mathrm{d}^2w}{\mathrm{d}t^2}$ o.e.	B1	1.1b
	$\frac{\mathrm{d}s}{\mathrm{d}t}=\frac{\mathrm{d}w}{\mathrm{d}t}-\frac{2}{5}\frac{\mathrm{d}^2w}{\mathrm{d}t^2}\Rightarrow\frac{\mathrm{d}w}{\mathrm{d}t}-\frac{2}{5}\frac{\mathrm{d}^2w}{\mathrm{d}t^2}=\frac{2}{5}w-90\mathrm{e}^{-t}$	M1	2.1
	$2\frac{\mathrm{d}^2w}{\mathrm{d}t^2}-5\frac{\mathrm{d}w}{\mathrm{d}t}+2w=450\mathrm{e}^{-t}$ *	A1*	1.1b
		(3)	
(b)	$2m^2-5m+2=0\Rightarrow m=...$	M1	3.4
	$m=2,\frac{1}{2}$	A1	1.1b
	$(w)=A\mathrm{e}^{\alpha t}+B\mathrm{e}^{\beta t}$	M1	3.4
	$(w)=A\mathrm{e}^{0.5t}+B\mathrm{e}^{2t}$	A1	1.1b
	PI: Try $w=k\mathrm{e}^{-t}\Rightarrow\frac{\mathrm{d}w}{\mathrm{d}t}=-k\mathrm{e}^{-t}\Rightarrow\frac{\mathrm{d}^2w}{\mathrm{d}t^2}=k\mathrm{e}^{-t}$ $2k\mathrm{e}^{-t}+5k\mathrm{e}^{-t}+2k\mathrm{e}^{-t}=450\mathrm{e}^{-t}\Rightarrow k=...$	M1	3.4
	$w=$ 'their C.F.' + $50\mathrm{e}^{-t}$ $\left(w=A\mathrm{e}^{0.5t}+B\mathrm{e}^{2t}+50\mathrm{e}^{-t}\right)$	A1ft	1.1b
		(6)	
(c)	$s=w-\frac{2}{5}\frac{\mathrm{d}w}{\mathrm{d}t}=A\mathrm{e}^{0.5t}+B\mathrm{e}^{2t}+50\mathrm{e}^{-t}-\frac{2}{5}\left(\frac{A}{2}\mathrm{e}^{0.5t}+2B\mathrm{e}^{2t}-50\mathrm{e}^{-t}\right)$	M1	3.4
	$s=\frac{4A}{5}\mathrm{e}^{0.5t}+\frac{B}{5}\mathrm{e}^{2t}+70\mathrm{e}^{-t}$	A1	1.1b
		(2)	
(d)	$65=A+B+50,\ 85=\frac{4A}{5}+\frac{B}{5}+70\Rightarrow A=...,B=...$ $\left(\text{NB } A=20\ \ B=-5\right)$	M1	3.3
	$w=0\Rightarrow 20\mathrm{e}^{0.5t}-5\mathrm{e}^{2t}+50\mathrm{e}^{-t}=0$	dM1	1.1b
	$\mathrm{e}^{3t}-4\mathrm{e}^{1.5t}-10(=0)$ or a multiple	A1	3.1a
	$\mathrm{e}^{1.5t}=\frac{4\pm\sqrt{4^2-4\times(1)(-10)}}{2}$	M1	1.1b
	$1.5t=\ln\left(\frac{4+\sqrt{56}}{2}\right)$	M1	2.3
	$T=\frac{2}{3}\ln\left(\frac{4+\sqrt{56}}{2}\right)=$ awrt 1.165	A1	3.2a
		(6)	

(e)	E.g. • Either population becomes negative which is not possible • When the white-clawed crayfish have died out, the model will not be valid	B1	3.5b
		(1)	
			(18 marks)

Notes

(a)

B1: Differentiates the first equation with respect to t correctly.

M1: Substitutes $\frac{\mathrm{d}s}{\mathrm{d}t}$ into their derivative.

A1*: Achieves the printed answer with no errors.

(b) **Note: All the mark except the final A1 are available if they use other variables.**

M1: Uses the model to form and solve the Auxiliary Equation.

A1: Correct roots of the AE.

M1: Uses the model to form the Complementary Function for their roots (they may be complex roots)

A1: Correct CF

M1: Chooses the correct form of the PI according to the model and uses a complete method to find the PI. Uses $w = ke^{-t}$ finds both $\frac{\mathrm{d}w}{\mathrm{d}t}$ and $\frac{\mathrm{d}^2w}{\mathrm{d}t^2}$ substitutes into the differential equation and find the value of k.

A1ft: Dependent on all three of the previous method marks. Following through on their CF only to give $w = \text{'their CF'} + 50e^{-t}$

(c)

M1: Substitutes into the first equation the answer for part (b) in place of w and the derivative of their (b) in place of $\frac{\mathrm{d}w}{\mathrm{d}t}$. If they rearrange to make S the subject first and make a slip but still substitutes for w and $\frac{\mathrm{d}w}{\mathrm{d}t}$ allow this mark.

A1: Correct simplified equation.

(d)

M1: Uses the initial conditions $t = 0$, $w = 65$ and $s = 85$ to form simulations equations and solves to find the values of their constants

dM1: Dependent on the previous method mark. Sets $w = 0$

A1: Processes the indices correctly to obtain a 3-term quadratic equation in terms of $e^{1.5t}$. It does not need to all be on one side and condone missing = 0.

M1: Solves their three-term quadratic (3TQ) to reach $e^{pt} = q$

M1: Correct use of logarithms to reach $pt = \ln q$ where $q > 0$ and rejects the other solution

A1: awrt 1.165

Note: the final 3 marks only can be implied by a correct answer following the correct 3-term quadratic equation in terms of $e^{1.5t}$

(e)

B1: Suggests a suitable limitation of the model, not valid when negative population

Any mention of other factors such as does not take into account e.g. other predictors, fishing, disease, lack of food etc is B0