

8. A scientist is studying the effect of introducing a population of white-clawed crayfish into a population of signal crayfish.

At time t years, the number of white-clawed crayfish, w , and the number of signal crayfish, s , are modelled by the differential equations

$$\begin{aligned}\frac{dw}{dt} &= \frac{5}{2}(w - s) \\ \frac{ds}{dt} &= \frac{2}{5}w - 90e^{-t}\end{aligned}$$

- (a) Show that

$$2 \frac{d^2w}{dt^2} - 5 \frac{dw}{dt} + 2w = 450e^{-t} \tag{3}$$

- (b) Find a general solution for the number of white-clawed crayfish at time t years.

(6)

- (c) Find a general solution for the number of signal crayfish at time t years.

(2)

The model predicts that, at time T years, the population of white-clawed crayfish will have died out.

Given that $w = 65$ and $s = 85$ when $t = 0$

- (d) find the value of T , giving your answer to 3 decimal places.

(6)

- (e) Suggest a limitation of the model.

(1)