Question	Scheme	Marks	AOs
6(i)	$n = 1$, LHS $= 2 \times 4 = 8$, RHS $= \frac{2}{3}(1)(1+1)(1+5) = 8$ So the result is true for $n = 1$	B1	2.2a
	Assume true for $n = k$ $$\sum_{r=1}^{k} 2r(r+3) = \frac{2}{3}k(k+1)(k+5)$$	M1	2.4
	$$\sum_{r=1}^{k+1} 2r(r+3) = \frac{2}{3}k(k+1)(k+5) + 2(k+1)(k+1+3)$$	M1	1.1b
	$$= \frac{2}{3}(k+1)[k(k+5) + 3(k+4)] \text{ or } \frac{2}{3}(k+1)[k^2 + 8k + 12]$$	A1	1.1b
	$$= \frac{2}{3}(k+1)(k+2)(k+6)$$	A1	2.1
	If **true for $n = k$** then **true for $n = k + 1$** and as it is **true for $n = 1$** the statement is **true for all** (positive integers) $\boldsymbol{n}$	A1	2.4
		(6)	
(ii)(a)	$n = 1, \begin{pmatrix} 5 & -8 \\ 2 & -3 \end{pmatrix}^1 = \begin{pmatrix} 5 & -8 \\ 2 & -3 \end{pmatrix}; \begin{pmatrix} 1+4(1) & -8(1) \\ 2(1) & 1-4(1) \end{pmatrix} = \begin{pmatrix} 5 & -8 \\ 2 & -3 \end{pmatrix}$ So the result is true for $n = 1$	B1	2.2a
	Assume true for $n = k$ $\begin{pmatrix} 5 & -8 \\ 2 & -3 \end{pmatrix}^k = \begin{pmatrix} 1+4k & -8k \\ 2k & 1-4k \end{pmatrix}$	M1	2.4
	$$\begin{pmatrix} 5 & -8 \\ 2 & -3 \end{pmatrix}^{k+1} = \begin{pmatrix} 1+4k & -8k \\ 2k & 1-4k \end{pmatrix}\begin{pmatrix} 5 & -8 \\ 2 & -3 \end{pmatrix}$$ or $$\begin{pmatrix} 5 & -8 \\ 2 & -3 \end{pmatrix}^{k+1} = \begin{pmatrix} 5 & -8 \\ 2 & -3 \end{pmatrix}\begin{pmatrix} 1+4k & -8k \\ 2k & 1-4k \end{pmatrix}$$	M1	1.1b
	$\begin{pmatrix} 1+4k & -8k \\ 2k & 1-4k \end{pmatrix}\begin{pmatrix} 5 & -8 \\ 2 & -3 \end{pmatrix}$ $$= \begin{pmatrix} 5(1+4k) - 16k & -8(1+4k) - 3(-8k) \\ 5(2k) + 2(1-4k) & -8(2k) - 3(1-4k) \end{pmatrix}$$ or $\begin{pmatrix} 5 & -8 \\ 2 & -3 \end{pmatrix}\begin{pmatrix} 1+4k & -8k \\ 2k & 1-4k \end{pmatrix}$ $$= \begin{pmatrix} 5(1+4k) - 8(2k) & 5(-8k) - 8(1-4k) \\ 2(1+4k) - 3(2k) & 2(-8k) - 3(1-4k) \end{pmatrix}$$	A1	1.1b
	$$\begin{pmatrix} 1+4(k+1) & -8(k+1) \\ 2(k+1) & 1-4(k+1) \end{pmatrix}$$	A1	2.1
	If **true for $n = k$** then **true for $n = k + 1$** and as it is **true for $n = 1$** the statement is **true for all** (positive integers) $\boldsymbol{n}$	A1	2.4
		(6)	

(ii)(b)	**Either** $\det(\mathbf{M}^n) = (1+4n)(1-4n) - (-8n)(2n) = 1$ **or** $\det(\mathbf{M}) = -15 + 16 = 1$	M1	2.1
	Either $\det(\mathbf{M}^n) = 1$ **or** $\det(\mathbf{M}^n) = (\det\mathbf{M})^n = 1$ Therefore $\det(\mathbf{M}^n)$ is **independent of *n***	A1	2.4
		(2)	
			(14 marks)

Notes:

(i)

B1: Shows that the result holds for $n = 1$. Must see substitution in the RHS and LHS and reach 8

M1: Makes a statement that assumes the result is true for some value of n

M1: Set up sum with assumed formula $2(k+1)(k+1+3)$

A1: Achieves a correct expression with a factor of $\frac{2}{3}(k+1)$

A1: Reaches a correct simplified expression with **no errors and the correct unsimplified expression with a factor of $\frac{2}{3}(k+1)$ seen previously.**

A1: Correct conclusion. This mark is dependent on all previous marks. It is gained by conveying the ideas of **all four bold** points either at the end of their solution or as a narrative in their solution

(ii) (a)

B1: Shows that the result holds for $n = 1$. Must see substitution in the RHS and reach $\begin{pmatrix} 5 & -8 \\ 2 & -3 \end{pmatrix}$

M1: Makes a statement that assumes the result is true for some value of n

M1: Set up a matrix multiplication of the assumed result multiplied by the original matrix, either way round

A1: Achieves a correct unsimplified matrix

A1: Reaches a correct simplified matrix with **no errors and the correct unsimplified matrix seen previously.**

A1: Correct conclusion. This mark is dependent on all previous marks. It is gained by conveying the ideas of **all four bold** points either at the end of their solution or as a narrative in their solution.

(ii) (b)

M1: Finds the determinant of matrix $\mathbf{M}^n$ or matrix $\mathbf{M}$

A1: Shows that $\det(\mathbf{M}^n) = 1$, therefore independent of n