

Question	Scheme	Marks	AOs
7(a)	Replaces $x^2 + y^2 \rightarrow r^2$ and $x \rightarrow r \cos \theta$ and $y \rightarrow r \sin \theta$	M1	2.1
	$r^4 = 6r^2 \cos \theta \sin \theta$ or $r^2 = 6 \cos \theta \sin \theta$	A1	1.1b
	$r^2 = 3 \sin 2\theta$ * cso	A1*	1.1b
		(3)	
(b)	Alternative 1 for the first three marks $y^2 = r^2 \sin^2 \theta = 3 \sin 2\theta \sin^2 \theta$ $2y \frac{dy}{d\theta} = \alpha \cos 2\theta \sin^2 \theta + \beta \sin 2\theta \sin \theta \cos \theta$	M1	3.1a
	$2y \frac{dy}{d\theta} = 6 \cos 2\theta \sin^2 \theta + 6 \sin 2\theta \sin \theta \cos \theta$	A1	1.1b
	$6 \cos 2\theta \sin^2 \theta + 6 \sin 2\theta \sin \theta \cos \theta = 0$ $6 \cos 2\theta \sin^2 \theta + 12 \sin^2 \theta \cos^2 \theta = 0$ $6 \sin^2 \theta (\cos 2\theta + 2 \cos^2 \theta) = 0$ $6 \sin^2 \theta (4 \cos^2 \theta - 1) = 0$ $\sin^2 \theta = 0 \Rightarrow \theta = \dots$ or $\cos^2 \theta = \frac{1}{4} \Rightarrow \theta = \dots$	M1	3.1a
	Alternative 2 for the first three marks $y = r \sin \theta = \sqrt{3} \sin \theta (\sin 2\theta)^{\frac{1}{2}}$ $\frac{dy}{d\theta} = \alpha \cos \theta (\sin 2\theta)^{\frac{1}{2}} + \beta (\sin 2\theta)^{-\frac{1}{2}} \cos 2\theta \cos \theta$	M1	3.1a
	$\frac{dy}{d\theta} = \sqrt{3} \cos \theta (\sin 2\theta)^{\frac{1}{2}} + \sqrt{3} (\sin 2\theta)^{-\frac{1}{2}} \cos 2\theta \cos \theta$	A1	1.1b
	$\sqrt{3} \cos \theta (\sin 2\theta)^{\frac{1}{2}} + \sqrt{3} (\sin 2\theta)^{-\frac{1}{2}} \cos 2\theta \cos \theta = 0$ $\sqrt{3} \cos \theta \sin 2\theta + \sqrt{3} \cos 2\theta \cos \theta = 0$ $\sqrt{3} \cos \theta \times 2 \sin \theta \cos \theta + \sqrt{3} (2 \cos^2 \theta - 1) \cos \theta = 0$ $\sqrt{3} \sin \theta (4 \cos^2 \theta - 1) = 0$ $\sin \theta = 0 \Rightarrow \theta = \dots$ or $\cos^2 \theta = \frac{1}{4} \Rightarrow \theta = \dots$	M1	3.1a
	Alternative 3 for the first three marks $(x^2 + y^2)^2 = 6xy \Rightarrow x^4 + 2x^2y^2 + y^4 = 6xy$ $4x^3 + \alpha xy^2 + \beta x^2y \frac{dy}{dx} + 4y^3 \frac{dy}{dx} = 6y + 6x \frac{dy}{dx}$	M1	3.1a
	$4x^3 + 4xy^2 + 4x^2y \frac{dy}{dx} + 4y^3 \frac{dy}{dx} = 6y + 6x \frac{dy}{dx}$	A1	1.1b
	$4x^3 + 4xy^2 + 4x^2y(0) + 4y^3(0) = 6y + 6x(0)$ $4x^3 + 4xy^2 = 6y$ $4r^3 \cos^3 \theta + 4r \cos \theta \times r^2 \sin^2 \theta = 6r \sin \theta$ $4r^3 \cos \theta [\cos^2 \theta + \sin^2 \theta] = 6r \sin \theta$ $4(3 \sin 2\theta) \cos \theta - 6 \sin \theta = 0$ $24 \sin \theta \cos^2 \theta - 6 \sin \theta = 0$ $6 \sin \theta [4 \cos^2 \theta - 1]$ $\sin \theta = 0 \Rightarrow \theta = \dots$ or $\cos^2 \theta = \frac{1}{4} \Rightarrow \theta = \dots$	M1	3.1a

For the final 6 marks			
Deduces that $\theta = \frac{\pi}{3}$ is required		A1	2.2a
Area bounded by the curve = $\frac{1}{2} \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 3 \sin 2\theta \, d\theta = [\delta \cos 2\theta]_{\frac{\pi}{3}}^{\frac{\pi}{2}} = \dots$ Note: $\delta = -\frac{3}{4}$		M1	1.1b
$\left\{ -\frac{3}{4} \cos \pi - \left(-\frac{3}{4} \cos \frac{2\pi}{3} \right) \right\} = \frac{3}{8}$		A1	1.1b
Area of triangle = $\frac{1}{2}xy = \frac{1}{2}r^2 \sin \theta \cos \theta$ $= \frac{1}{2} \times 3 \sin \frac{2\pi}{3} \sin \frac{\pi}{3} \cos \frac{\pi}{3} = \dots$		M1	1.1b
Finds the required area = area of triangle – area bounded by the curve $\left\{ = \frac{9}{16} - \frac{3}{8} \right\}$		ddM1	3.1a
$= \frac{3}{16}$		A1	1.1b
		(9)	

(12 marks)

Notes:

- (a)**
M1: Replaces all Cartesian variables with polar variables
A1: A correct unsimplified polar equation
A1*: Correct polar equation achieved from correct work.

- (b) Alternatives 1 and 2**
M1: Writes y^2 or y in terms of trig and differentiates (implicitly) to the correct form
A1: Fully correct differentiation
M1: Uses correct trig work to solve $\frac{dy}{d\theta} = 0$ to achieve a value for θ

- (b) Alternative 3**
M1: Differentiates implicitly the Cartesian equation to the correct form
A1: Fully correct differentiation
M1: Uses correct polar equation and correct trig work to solve $\frac{dy}{dx} = 0$ to achieve a value for θ

- (b) For the last 6 marks**
A1: Deduces that $\theta = \frac{\pi}{3}$
M1: Applies area = $\frac{1}{2} \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} r^2 \, d\theta$, integrates to a correct form and applies their lower limit and upper limits of $\frac{\pi}{2}$ correctly
A1: Correct area under the curve
M1: Correct method to find the area of triangle = $\frac{1}{2}xy$
ddM1: Dependent of previous two method marks. Fully correct method to find the required area.
 Finds the required area = area of triangle – area bounded by the curve
A1: Correct answer