

3.

$$\mathbf{A} = \begin{pmatrix} k & -2 \\ 1-k & k \end{pmatrix} \quad \text{where } k \text{ is a constant}$$

(a) Show that the matrix \mathbf{A} is non-singular for all values of k .

(2)

A transformation $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is represented by the matrix \mathbf{A} .

The point P has position vector $\begin{pmatrix} a \\ 2a \end{pmatrix}$ relative to an origin O .

The point Q has position vector $\begin{pmatrix} 7 \\ -3 \end{pmatrix}$ relative to O .

Given that the point P is mapped onto the point Q under T ,

(b) determine the value of a and the value of k .

(3)

Given that, for a different value of k , T maps the line $y = 2x$ onto itself,

(c) determine this value of k .

(3)