6 (i) Prove by induction that for $n \in \mathbb{Z}^+$

$$2 \times 4 + 4 \times 5 + 6 \times 6 + \ldots + 2n(n+3) = \frac{2}{3}n(n+1)(n+5)$$

(6)

(ii) Given that

$$\mathbf{M} = \begin{pmatrix} 5 & -8 \\ 2 & -3 \end{pmatrix}$$

(a) Prove by induction that for $n \in \mathbb{Z}^+$

$$\mathbf{M}^n = \begin{pmatrix} 1+4n & -8n \\ 2n & 1-4n \end{pmatrix}$$

(6)

(b) Show that $\det(\mathbf{M}^n)$ is independent of n.

(2)