8.

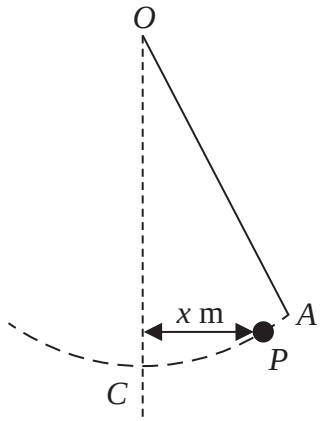

Figure 2

A child plays on a rope swing.
One end of the rope is attached to a tree and the child sits on a large knot at the other end of the rope.
The child swings back and forth in a vertical plane.

The rope is modelled as a light and inextensible string. The child is modelled as a particle.

Figure 2 represents the child and the rope swing. The rope is attached to the tree at the point O and the point C is vertically below O. The point P represents the child.

The horizontal displacement of P from the line OC at time t seconds ($t \geqslant 0$) is x metres, as shown in Figure 2.

The motion of P is modelled by the differential equation

$$\ddot{x} + 2\dot{x} + \lambda x = 0$$

where λ is a positive constant.

The child is initially at rest, at the point A, with a horizontal displacement of 1.5 m from the line OC.

Given that the initial horizontal acceleration of the child is $-7.5\,\text{m s}^{-2}$

(a) show that $\lambda = 5$

(2)

Using the model,

(b) find an expression for the horizontal displacement of the child at time t.

(7)

Given that, when $t = 4.5$, the child is vertically below O,

(c) evaluate the model explaining your reasoning.

(2)

(d) Suggest one refinement for the model.

(1)