Question	Scheme	Marks	AOs
2(a)	E.g. • Because the interval being integrated over is unbounded • Accept because the upper limit is infinity • Accept because a limit is required to evaluate it	B1	2.4
		(1)	
(b)	$\frac{1}{x(2x+5)} = \frac{A}{x} + \frac{B}{2x+5} \Rightarrow A = ..., B = ...$	M1	3.1a
	$\frac{1}{x(2x+5)} = \frac{1}{5x} - \frac{2}{5(2x+5)}$	A1	1.1b
	$\int \frac{1}{5x} - \frac{2}{5(2x+5)}\,dx = \frac{1}{5}\ln x - \frac{1}{5}\ln(2x+5)$	A1ft	1.1b
	$\frac{1}{5}\ln x - \frac{1}{5}\ln(2x+5) = \frac{1}{5}\ln\frac{x}{(2x+5)}$	M1	2.1
	$\lim_{x\to\infty}\left\{\frac{1}{5}\ln\frac{x}{2x+5}\right\} = \frac{1}{5}\ln\frac{1}{2}$	B1	2.2a
	$\Rightarrow \int_1^\infty \frac{1}{x(2x+5)}\,dx = \frac{1}{5}\ln\frac{1}{2} - \frac{1}{5}\ln\frac{1}{7} = \frac{1}{5}\ln\frac{7}{2}$	A1	1.1b
		(6)	
			(7 marks)

Notes

(a)

B1: For a suitable explanation with no contrary reasoning. Technically this should refer to the interval being unbounded, but this is unlikely to be seen. Accept “Because the upper limit is infinity”. Do not award if there are erroneous statements e.g. referring to as $x = 0$ the integrand is not defined. Do not accept “because one of the limits is undefined” unless they state they mean ∞. Do not accept “it is undefined when $x = \infty$” without reference to “it” being the upper limit.

(b)

M1: Selects the correct form for partial fractions and proceeds to find values for *A* and *B*

A1: Correct constants or partial fractions

A1ft: $\int \frac{p}{x} + \frac{q}{2x+5}\,dx = p\ln x + \frac{q}{2}\ln(2x+5)$ Note that $\frac{1}{5}\ln 5x - \frac{1}{5}\ln(10x+25)$ is correct.

M1: Combines logs correctly. May see $-\frac{1}{5}\ln\left(\frac{2x+5}{x}\right) = -\frac{1}{5}\ln\left(2+\frac{5}{x}\right)$

B1: Correct upper limit for $x \to \infty$ by recognising the dominant terms. (Simply replacing *x* with ∞ scores B0)

A1: Deduces the correct value for the improper integral in the correct form

Question	Scheme	Marks	AOs
(b) **Way 2**	$\dfrac{1}{x(2x+5)}=\dfrac{1}{2\left(x^2+\frac{5}{2}x\right)}=\dfrac{1}{2}\times\dfrac{1}{\left(x+\frac{5}{4}\right)^2-\frac{25}{16}}$	M1 A1	3.1a 1.1b
	$\displaystyle\int\frac{1}{x(2x+5)}\,\mathrm{d}x=\frac{1}{2}\times\frac{2}{5}\ln\left\lvert\frac{x+\frac{5}{4}-\frac{5}{4}}{x+\frac{5}{4}+\frac{5}{4}}\right\rvert=\frac{1}{5}\ln\left\lvert\frac{2x}{2x+5}\right\rvert$	M1 A1ft	2.1 1.1b
	$\displaystyle\lim_{x\to\infty}\left\{\frac{1}{5}\ln\frac{2x}{2x+5}\right\}=\frac{1}{5}\ln\frac{2}{2}=0$	B1	2.2a
	$\displaystyle\Rightarrow\int_1^{\infty}\frac{1}{x(2x+5)}\,\mathrm{d}x=0-\frac{1}{5}\ln\frac{2}{7}=\frac{1}{5}\ln\frac{7}{2}$	A1	1.1b
		(6)	

Notes

Note the method marks as MAMABA, and should be entered in this order on ePEN.

M1: Expands the denominator and completes the square.

A1: Correct expression

M1: For $\dfrac{1}{(x+p)^2-a^2}\to k\ln\left\lvert\dfrac{x+p-a}{x+p+a}\right\rvert$

A1ft: $\dfrac{1}{2}\dfrac{1}{(x+a)^2-a^2}\to\dfrac{1}{2a}\ln\left\lvert\dfrac{x}{x+2a}\right\rvert$ with their a (may be simplified as in scheme).

B1: Correct upper limit for $x\to\infty$ by recognising the dominant terms. (Simply replacing x with ∞ scores B0) Note in this method the upper limit evaluates to zero.

A1: Deduces the correct value for the improper integral in the correct form. Accept $-\dfrac{1}{5}\ln\dfrac{2}{7}$