

Question	Scheme	Marks	AOs
3	$3(1 - \sin \theta) = 1 + \sin \theta \Rightarrow \sin \theta = \frac{1}{2} \Rightarrow \theta = \dots$	M1	3.1a
	$\theta = \frac{\pi}{6} \left(\text{or } \frac{5\pi}{6} \right)$	A1	1.1b
	Use of $\frac{1}{2} \int (1 + \sin \theta)^2 d\theta$ or $\frac{1}{2} \int \{3(1 - \sin \theta)\}^2 d\theta$	M1	1.1a
	$\left(\frac{1}{2}\right) \int \left[(1 + \sin \theta)^2 - 9(1 - \sin \theta)^2 \right] d\theta$ $= \left(\frac{1}{2}\right) \int \left[1 + 2\sin \theta + \sin^2 \theta - 9 + 18\sin \theta - 9\sin^2 \theta \right] d\theta$ <p style="text-align: center;">or</p> $\int (1 + \sin \theta)^2 d\theta = \int (1 + 2\sin \theta + \sin^2 \theta) d\theta \text{ and}$ $\int 9(1 - \sin \theta)^2 d\theta = 9 \int (1 - 2\sin \theta + \sin^2 \theta) d\theta$	M1 A1	2.1 1.1b
	$\int \sin^2 \theta d\theta = \frac{1}{2} \int (1 - \cos 2\theta) d\theta \Rightarrow$ $\int \left[(1 + \sin \theta)^2 - 9(1 - \sin \theta)^2 \right] d\theta = 2\sin 2\theta - 12\theta - 20\cos \theta$	M1 A1	3.1a 1.1b
	$A = \frac{1}{2} \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \left[(1 + \sin \theta)^2 - 9(1 - \sin \theta)^2 \right] d\theta$ <p style="text-align: center;">or</p> $A = 2 \times \frac{1}{2} \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \left[(1 + \sin \theta)^2 - 9(1 - \sin \theta)^2 \right] d\theta$ $= \frac{1}{2} \left\{ (-\sqrt{3} - 10\pi + 10\sqrt{3}) - (\sqrt{3} - 2\pi - 10\sqrt{3}) \right\} = \dots$	DM1	3.1a
	$= 9\sqrt{3} - 4\pi$	A1	1.1b

(9)

(9 marks)

Notes

M1: Realises that the angles at the intersection are required and solves $C_1 = C_2$ to obtain a value for θ

A1: Correct value for θ . Must be in radians – if given in degrees you may need to check later to see if they convert to radians before substitution.

M1: Evidence selecting the correct polar area formula on either curve

M1: Fully expands both expressions for r^2 either as parts of separate integrals or as one complete integral. (Can be scored from incorrect polar area formula, e.g. missing the $\frac{1}{2}$)

A1: Correct expansions for both curves (may be unsimplified)

M1: Selects the correct strategy by applying the correct double angle identity in order to reach an integrable form and attempting the integration of at least one of the curves.

A1: Correct integration (of both integrals if done separately),

FYI: If done separately the correct integrals are

$$\int (1 + \sin \theta)^2 d\theta = \theta - 2 \cos \theta + \frac{1}{2} \left(\theta - \frac{1}{2} \sin 2\theta \right) = \frac{3}{2} \theta - 2 \cos \theta - \frac{1}{4} \sin 2\theta \quad \text{and}$$

$$\int 9(1 - \sin \theta)^2 d\theta = 9\theta + 18 \cos \theta + \frac{9}{2} \left(\theta - \frac{1}{2} \sin 2\theta \right) = \frac{27}{2} \theta + 18 \cos \theta - \frac{9}{4} \sin 2\theta$$

DM1: Depends on all previous M's. For a fully correct strategy with appropriate limits correctly applied to their integral or integrals and terms combined if necessary. Make sure that if limits of

$\frac{\pi}{6}$ and $\frac{\pi}{2}$ are used that the area is doubled as part of the strategy.

A1: Correct area