Question	Scheme	Marks	AOs
5(a)	$\frac{d^2x}{dt^2} = -5\frac{dx}{dt} + 10\frac{dy}{dt}$ oe e.g. $\frac{dy}{dt} = \frac{1}{10}\left(\frac{d^2x}{dt^2} + 5\frac{dx}{dt}\right)$	B1	1.1b
	$\frac{d^2x}{dt^2} = -5\frac{dx}{dt} + 10(-2x+3y-4)$ $= -5\frac{dx}{dt} - 20x + \frac{30}{10}\left(\frac{dx}{dt} + 5x + 30\right) - 40$ Or $\frac{1}{10}\left(\frac{d^2x}{dt^2} + 5\frac{dx}{dt}\right) = -2x + \frac{3}{10}\left(30 + 5x + \frac{dx}{dt}\right) - 4$	M1	2.1
	$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = 50$ *	A1*	1.1b
		(3)	
(b)	$m^2 + 2m + 5 = 0 \Rightarrow m = \ldots$	M1	3.4
	$m = -1 \pm 2i$	A1	1.1b
	$m = \alpha \pm \beta i \Rightarrow x = e^{\alpha t}(A\cos\beta t + B\sin\beta t) = \ldots$	M1	3.4
	$x = e^{-t}(A\cos 2t + B\sin 2t)$	A1	1.1b
	PI: Try $x = k \Rightarrow 5k = 50 \Rightarrow k = 10$	M1	3.4
	$GS: x = e^{-t}(A\cos 2t + B\sin 2t) + 10$	A1ft	1.1b
		(6)	
(c)	$\frac{dx}{dt} = e^{-t}(2B\cos 2t - 2A\sin 2t) - e^{-t}(A\cos 2t + B\sin 2t)$	B1ft	1.1b
	$(y =)\frac{1}{10}\left(\frac{dx}{dt} + 5x + 30\right) = \ldots$	M1	3.4
	$y = \frac{1}{10}e^{-t}\left((4A+2B)\cos 2t + (4B-2A)\sin 2t\right) + 8$	A1	1.1b
		(3)	
(d)	$t = 0, x = 2 \Rightarrow 2 = A + 10 \Rightarrow A = -8$	M1	3.1b
	$t = 0, y = 5 \Rightarrow 5 = \frac{1}{10}(2B - 32) + 8 \Rightarrow B = 1$	M1	3.3
	$x = e^{-t}(\sin 2t - 8\cos 2t) + 10$	A1	2.2a
	$y = e^{-t}(2\sin 2t - 3\cos 2t) + 8$	A1	2.2a
		(4)	
(e)	E.g When $t > 8$, the amount of compound X and the amount of compound Y remain (approximately) constant at 10 and 8 respectively, which suggests that the chemical reaction has stopped. This supports the scientist's claim.	B1	3.5a
		(1)	
			(17 marks)

Notes
(a) B1: Differentiates the first equation with respect to t correctly. May have rearranged to make y the subject first. The dot notation for derivatives may be used. M1: Uses the second equation to eliminate y to achieve an equation in x, $\frac{\mathrm{d}x}{\mathrm{d}t}, \frac{\mathrm{d}^2x}{\mathrm{d}t^2}$. A1*: Achieves the printed answer with no errors. (b) M1: Uses the model to form and attempts to attempts to solve the auxiliary equation (Accept a correct equation followed by two values for m as an attempt to solve.) A1: Correct roots of the AE M1: Uses the model to form the complementary function. Must be in terms of t only (not x) A1: Correct CF M1: Chooses the correct form of the PI according to the model and uses a complete method to find the PI A1ft: Combines their CF (which need not be correct) with the correct PI to give x in terms of t so look for x = their CF + 10 (c) B1ft: Correct differentiation of their x. Follow through their $\mathrm{e}^{\alpha t}\left(A\cos\beta t + B\sin\beta t\right)$ M1: Uses the model and their answer to part (b) to find an expression for y in terms of t A1: Correct equation. Mark the final answer but there is not need for terms to be gathered but must have $y = \ldots$. (d) M1: Realises the need to use the initial conditions in the equation for x M1: Realises the need to use the initial conditions in the equation for y to find both unknown constants - must have equations from which both unknowns can be found. Alternatively, a complete method using $\frac{\mathrm{d}x}{\mathrm{d}t}$ to find the second constant is made. A1: Deduces the correct equation for x A1: Deduces the correct equation for y. For this equation constants should have been gathered. (e) B1: Allow for any appropriate comment with valid supporting reason. They must have equations of the correct form from (d). The coefficients may be incorrect, but they must have positive limits for each of x and y. Both x and y should be considered (see below for exception), and a reason and some comment about the suitability of the model made (though you may allow implicit conclusions). E.g. • for values of $t > 8$, the amounts of compounds X and Y present settle at 10 and 8 without really varying, which supports the claim. • $\frac{\mathrm{d}x}{\mathrm{d}t} \approx 0$ and $\frac{\mathrm{d}y}{\mathrm{d}t} \approx 0$ when $t = 8$, so neither are changing, which supports the claim. • As t gets large x and y tend to limits to 10 and 8 neither will be zero, hence the claim is not supported. • $x = 10.0$ (awrt) and $y = 8.00$ (awrt) when $t = 8$, since neither is zero it is likely the reaction is still continuing so the claim is not supported. Exception: Allow a reason that states the model assumes that the reaction continues indefinitely, so the claim is not supported. (The reaction stopping would require a change in the model.)

Do NOT allow an answer that only considers x or y. E.g. $x = 10$ when $t = 8$ so the model is not supported is B0 since there is no consideration that y may be zero and hence end the reaction.

Alt for (c) and (d) restarting:

B1: Correct second order equation for y formed: $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = 40$

M1: Full method to obtain the general solution: they may recognise the similarity to the equation in x and jump straight to finding the PI, or may form the aux equation etc again, but look for an attempt that combines a (correctly formed) CF and a PI. For this mark allow if the constants used are the same as those for the equation in x.

A1: Correct solution for y with different constants than those for x, though allow recovery if they realise in (d) that they need different constants.

For (d)

M1: As main scheme, allow for using the initial conditions in one equation to make a start finding the constants.

M1: For a full method to obtain all four constants – if the same constant were used for both equations in (c) (inconsistently) then this mark cannot be scored. A full method here would, for instance, require finding $\frac{dx}{dt}$ and using this along with the given initial equations and initial conditions to find the second constants for each equation.

A1: One correct equation with SC of being qualified by the first M only if a full method to find both constants for just one equation is made (so M1M0A1A0 is possible in this case).

A1: Both equations correct.