Question	Scheme	Marks	AOs
6(i)	When $n=1$, $\sum_{r=1}^{1}(3r+1)(r+2)=4\times3=12$ $1(1+2)(1+3)=12$ (so the statement is true for $n=1$)	B1	2.2a
	Assume true for $n=k$ so $\sum_{r=1}^{k}(3r+1)(r+2)=k(k+2)(k+3)$	M1	2.4
	$\sum_{r=1}^{k+1}(3r+1)(r+2)=k(k+2)(k+3)+(3k+4)(k+3)$	M1	2.1
	$=(k+3)(k^2+5k+4)$	A1	1.1b
	$\sum_{r=1}^{k+1}(3r+1)(r+2)=(k+1)(k+3)(k+4)$	A1	1.1b
	$\sum_{r=1}^{k+1}(3r+1)(r+2)=(\underline{k+1})(\underline{k+1}+2)(\underline{k+1}+3)$ If the statement is <u>true for $n=k$ then</u> it has been shown <u>true for $n=k+1$</u> and as it is <u>true for $n=1$</u>, the statement <u>is true for all (positive integers) n.</u>	A1	2.4
		(6)	
(ii) Way 1	When $n=1$, $4^1+5^1+6^1=15$ so the statement is true for $n=1$	B1	2.2a
	Assume true for $n=k$ so $4^k+5^k+6^k$ is divisible by 15	M1	2.4
	$f(k+2)=4^{k+2}+5^{k+2}+6^{k+2}$	M1	2.1
	$=16\times4^k+16\times5^k+16\times6^k+9\times5^k+20\times6^k$ $=16f(k)+45\times5^{k-1}+120\times6^{k-1}$	A1 A1	1.1b 1.1b
	E.g As 15 divides f(k), 45 and 120, so 15 divides f(k+1). <u>If true for $n=k$</u> then <u>true for $n=k+2$</u>, <u>true for $n=1$</u> so <u>true for all positive odd integers n</u>	A1	2.4
		(6)	
(ii) Way 2	When $n=1$, $4^1+5^1+6^1=15$ so the statement is true for $n=1$	B1	2.2a
	Assume true for $n=k$ so $4^k+5^k+6^k$ is divisible by 15	M1	2.4
	$f(k+2)-f(k)=4^{k+2}+5^{k+2}+6^{k+2}-4^k-5^k-6^k$	M1	2.1
	$=15\times4^k+24\times5^k+35\times6^k$ $=15f(k)+45\times5^{k-1}+120\times6^{k-1}$	A1	1.1b
	$f(k+2)=16f(k)+45\times5^{k-1}+120\times6^{k-1}$	A1	1.1b
	E.g $f(k+2)=16f(k)+15(3\times5^{k-1}+8\times6^{k-1})$ so <u>if true for $n=k$</u> then <u>true for $n=k+2$</u>, <u>true for $n=1$</u> so <u>true for all positive odd integers n</u>	A1	2.4
		(6)	

(ii) Way 3	When $n = 1,\ 4^1 + 5^1 + 6^1 = 15$ so the statement is true for $n = 1$	B1	2.2a
	Assume true for $n = 2k+1$ so $\text{f}(2k+1) = 4^{2k+1} + 5^{2k+1} + 6^{2k+1}$ is divisible by 15	M1	2.4
	$\text{f}\left(2k+3\right) = 4^{2k+3} + 5^{2k+3} + 6^{2k+3}$ or $\text{f}\left(2k+3\right) - \text{f}\left(2k+1\right) = 4^{2k+3} + 5^{2k+3} + 6^{2k+3} - 4^{2k+1} - 5^{2k+1} - 6^{2k+1}$	M1	2.1
	$\text{f}(2k+3) = 16\times 4^{2k+1} + 25\times 5^{2k+1} + 36\times 6^{2k+1}$ $= 16\left(4^{2k+1} + 5^{2k+1} + 6^{2k+1}\right) + 9\times 5^{2k+1} + 20\times 6^{2k+1}$ OR $\text{f}(2k+3) - \text{f}(2k+1) = 16\times 4^{2k+1} + 25\times 5^{2k+1} + 36\times 6^{2k+1} - 4^{2k+1} - 5^{2k+1} - 6^{2k+1}$ $= 15\times 4^{2k+1} + 120\times 5^{2k} + 210\times 6^{2k}$	A1	1.1b
	$\text{f}(2k+3) = 16\left(4^{2k+1} + 5^{2k+1} + 6^{2k+1}\right) + 45\times 5^{2k} + 120\times 6^{2k}$ OR $\text{f}\left(2k+3\right) = \text{f}\left(2k+1\right) + 15\times 4^{2k+1} + 120\times 5^{2k} + 210\times 6^{2k}$	A1	1.1b
	$\text{f}(2k+3) = 16\left(4^{2k+1} + 5^{2k+1} + 6^{2k+1}\right) + 15\left(3\times 5^{2k} + 8\times 6^{2k}\right)$ OR $\text{f}\left(2k+3\right) = \text{f}\left(2k+1\right) + 15\left(4^{2k+1} + 8\times 5^{2k} + 14\times 6^{2k}\right)$ and <u>If true for $n = 2k+1$</u> then <u>true for $n = 2k+3$</u>, <u>true for $n = 1$</u> so <u>true for all positive odd integers n</u>	A1	2.4
		(6)	

(12 marks)

Notes

(i)
B1: Shows the statement is true for $n = 1$ by evaluating **both** sides. There is no need for statement "hence true for $n = 1$" for this mark but if they never state this the final A will be forfeited.
Look for a minimum of $4\times 3 = 12$ for the LHS and $1\times 3\times 4$ for the RHS. If only one side is explicitly evaluated, it is B0, but all other marks may be gained.
M1: Makes an assumption statement that assumes the result is true for $n = k$
M1: Makes the inductive step by attempting to add the $(k+1)^{\text{th}}$ term to the assumed result. Attempts at using the standard summation formulae score M0, as the question requires induction.
A1: Correct expression with at least one correct linear factor
A1: Obtains a fully correct factorised expression. May be as in scheme or in terms of $k + 1$.
A1: Correct complete conclusion with all ideas conveyed at the end or as a narrative and the sum to k+1 expressed in terms of $k + 1$ (or with the expression in term of $k + 1$ stated earlier – it must be seen at some stage). Allow slips in notation if the intent is correct. Depends on all except the B mark, though an attempt at checking the $n = 1$ case must have been made.
(ii) **Way 1**
B1: Shows that f(1) = 15
M1: Makes a statement that assumes the result is true for some value of n (Assume (true for) $n = k$ is sufficient – note that this may be recovered in their conclusion if they say e.g. if true for $n = k$ then … etc.)
M1: Attempts $\text{f}(k + 2)$

A1: Correctly obtains 16f(k) **or** $45\times5^{k-1}+120\times6^{k-1}$
A1: Reaches a correct expression for f(k + 2) in terms of f(k)
A1: Correct conclusion, including explanation that all terms on RHS are divisible by 15, or writing terms each explicitly as multiples of 15. This mark is dependent on all previous marks apart from the B mark. It is gained by conveying the ideas of **all** four underlined points **either** at the end of their solution **or** as a narrative in their solution. Do not allow "true for all n" where n represents natural numbers, as this is incorrect.

Way 2
B1: Shows that f(1) = 15
M1: Makes a statement that assumes the result is true for some value of n (Assume (true for) $n = k$ is sufficient – note that this may be recovered in their conclusion if they say e.g. if true for $n = k$ then … etc.)
M1: Attempts f(k + 2) – f(k) or equivalent work
A1: Achieves a correct expression for f(k + 2) – f(k) in terms of f(k)
A1: Reaches a correct expression for f(k + 2) in terms of f(k)
A1: Correct conclusion, including explanation that all terms on RHS are divisible by 15, or writing terms each explicitly as multiples of 15. This mark is dependent on all previous marks apart from the B mark. It is gained by conveying the ideas of **all** four underlined points **either** at the end of their solution **or** as a narrative in their solution. Do not allow "true for all n" where n represents natural numbers, as this is incorrect.

Way 3
B1: Shows that f(1) = 15
M1: Makes a statement that assumes the result is true for some odd value of n (Assume (true for) $n = 2k + 1$ is sufficient (or may use $2k-1$ – note that this may be recovered in their conclusion if they say e.g. if true for $n = k$ then … etc.)
M1: Attempts f(2k+3) (like Way 1), or f(2k +3) – f(2k+1) or equivalent work (like Way 2) It must be the correct f(2k+1) used, in the latter case (subtracting $4^k+5^k+6^k$ instead of $4^{2k+1}+5^{2k+1}+6^{2k+1}$ is M0)
A1: Reaches $16\left(4^{2k+1}+5^{2k+1}+6^{2k+1}\right)+9\times5^{2k+1}+20\times6^{2k+1}$ or suitable equivalent OR achieves a correct expression for f(2k +3) – f(2k+1) in terms of f(2k+1) where factors of 15 are apparent.
A1: Reaches $16\left(4^{2k+1}+5^{2k+1}+6^{2k+1}\right)+45\times5^{2k}+120\times6^{2k}$ or a suitable equivalent OR a correct expression for f(2k + 3) of form $A\mathrm{f}(2k+1)+15(...)$ (though the 15(…) could be separate multiples of 15 where the 15 need not yet be extracted).
A1: Correct conclusion, including explanation that all terms on RHS are divisible by 15, or writing terms each explicitly as multiples of 15. This mark is dependent on all previous marks apart from the B mark. It is gained by conveying the ideas of **all** four underlined points **either** at the end of their solution **or** as a narrative in their solution. Do not allow "true for all n" where n represents natural numbers, as this is incorrect, but if n represents odd numbers than allow.
Note: $4^{2k}=16^k$ etc may be used throughout.
Note: Way 3 should apply to cases where the question is rephrased in terms of $4^{2n+1}+5^{2n+1}+6^{2n+1}$ for all n (rather than odd n).
Accept use of alternative equivalent language throughout.